Uwe Waldmann December 11, 2019

Tutorials for "Automated Reasoning"
Exercise sheet 8

Exercise 8.1: *(3+3+3 P)*
Suppose that the atom ordering $\succ$ compares ground atoms by comparing lexicographically first the predicate symbols $(P > Q > R)$, then the size of the first argument, and then the size of the second argument (if present). If at least one of the two atoms to be compared is non-ground, $\succ$ compares only the predicate symbols.

Let N be the following set of clauses:

$$P(f(x), x) \vee R(b, b) \tag{1}$$
$$\neg P(b, x) \vee \neg P(x, b) \vee Q(x) \tag{2}$$
$$Q(f(b)) \vee \neg Q(b) \vee R(f(x), b) \tag{3}$$
$$Q(b) \vee \neg R(f(x), f(x)) \tag{4}$$
$$\neg Q(x) \vee R(x, x) \tag{5}$$

(a) Which literals are maximal in the clauses of N?

(b) Which $Res_{sel}^{\succ}$-inferences are possible if *sel* selects no literals?

(c) Define a selection function *sel* such that N is saturated under $Res_{sel}^{\succ}$.

Exercise 8.2: *(3+3 P)*
Let N be the following set of ground clauses:

$$\neg P_3 \vee P_1 \vee P_1 \tag{1}$$
$$\neg P_2 \vee P_1 \tag{2}$$
$$P_4 \vee P_4 \tag{3}$$
$$P_3 \vee \neg P_2 \tag{4}$$
$$P_4 \vee P_3 \tag{5}$$

(a) Find a total atom ordering $\succ$ such that both clause (2) and (5) are redundant w.r.t. N.

(b) Prove that there is no atom ordering such that clause (4) is redundant w.r.t. N.

Exercise 8.3: (*5 P*)
Prove that it is undecidable whether a clause C is redundant w. r. t. a set of clauses N. (You may use the fact that the satisfiability of a set of first-order clauses is undecidable.)

Challenge Problem: (*6 Bonus Points*)
Let $\Sigma = (\Omega, \Pi)$ be a signature such that Ω contains at least one constant symbol. A Σ-algebra $\mathcal{A}$ is called *term-generated*, if every $a \in U_{\mathcal{A}}$ is term-generated. Prove that a closed prenex formula without existential quantifiers (possibly including equality) has a model if and only if it has a term-generated model.

Submit your solution in lecture hall E1.3, Room 001 during the lecture on December 18. Please write the time of your tutorial group (Mon or Tue) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.