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Tutorials for "Automated Reasoning"
Exercise sheet 7

Exercise 7.1: (*4 P*)
Prove the following statement: If N is a set of propositional formulas and C is a propositional formula such that $N \models C$, then there exists a finite subset $M \subseteq N$ such that $M \models C$.

Exercise 7.2: (*3+3 P*)
Using the (i) standard and the (ii) polynomial unification rules, compute most general unifiers of $P(g(x_1, g(f(x_3), x_3)), g(h(x_4), x_3))$ and $P(g(x_2, x_2), g(x_3, h(x_1)))$, if they exist.

Exercise 7.3: (*5 P*)
Prove that the relation $\Rightarrow_{PU}$ (rule-based polynomial unification) is terminating. Hint: The first component of the lexicographic combination of orderings used to prove termination of $\Rightarrow_{SU}$ can be kept, but the second one cannot, due to the last rule for $\Rightarrow_{PU}$.

Exercise 7.4: (*5 P*)
Compute $Res(N)$ for the following set N of (implicitly universally quantified) clauses:

$$
\begin{array}{lc}
(1) & P(x, x) \vee P(h(x', b), h(c, x'')) \\
(2) & \neg P(y, f(y)) \vee Q(g(y)) \\
(3) & \neg Q(z) \vee P(d, z)
\end{array}
$$

(You may write down the side computations for the mgu's, but you don't have to. Do not compute $Res^*(N)$!)

Challenge Problem: (*6 Bonus Points*)
Prove part (ii) of Prop. 3.24: If $\sigma \leq \tau$ and $\tau \leq \sigma$, then there exist variable renamings δ and δ' (i.e., *bijective* substitutions mapping variables to variables), so that $x\sigma\delta = x\tau$ and $x\tau\delta' = x\sigma$ for every x in X. (Note: $\{x \mapsto y\}$ is *not* a bijective substitution!)

Submit your solution in lecture hall E1.3, Room 001 during the lecture on December 11. Please write the time of your tutorial group (Mon or Tue) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.