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Tutorials for "Automated Reasoning"
Exercise sheet 6

Exercise 6.1: *(4 P)*
Transform the first-order formula

$$F \;=\; \forall x\, \exists y\, \exists z \Big(\big(P(x) \land R(x,y)\big) \lor \neg \forall w\, Q(z,w) \Big)$$

into clause normal form using the improved algorithm from Section 3.6. (There are no subformulas in F for which one should introduce a definition.)

Exercise 6.2: *(6 P)*
Let $\Sigma = (\Omega, \Pi)$ with $\Omega = \{b/0,\, c/0,\, d/0\}$ and $\Pi = \{P/1,\, Q/0\}$. Are the following statements correct? Give a brief explanation.

(1) The formula $\forall x\, P(x)$ has infinitely many Σ-models.

(2) The formula $P(b) \land P(c) \land \neg P(d)$ has a Σ-model with a two-element universe.

(3) Every Σ-model of $\neg P(b) \land \neg P(c)$ has a finite universe.

(4) Every Σ-model of $P(b) \land P(c) \land P(d)$ is a model of $\forall x\, P(x)$.

(5) Every Herbrand model over Σ of $P(b) \land P(c) \land P(d)$ is a model of $\forall x\, P(x)$.

(6) The formula $P(b) \land P(c) \land \neg P(d)$ has exactly two Herbrand models over Σ.

Exercise 6.3: *(2+2 P)*
Let N be the following set of ground clauses:

$$\{\, P \lor Q,\; P \lor \neg Q,\; \neg P \lor Q,\; \neg P \lor \neg Q \,\}$$

(i) Show that $N \vdash_{Res} \bot$, that is, derive $\bot$ from N using the Resolution and the Positive Factorization rule.

(ii) Why is it impossible to derive the empty clause from these clauses without using factorization?

Exercise 6.4: *(3+3 P)*
Let N be the set containing the following ground clauses:

$$
\begin{aligned}
C_1 &= P(a) \vee \neg Q(a) \vee \neg Q(b) \\
C_2 &= P(a) \vee P(a) \vee Q(b) \\
C_3 &= P(b) \vee Q(a) \vee Q(b) \\
C_4 &= Q(a) \vee Q(b) \vee Q(b) \\
C_5 &= \neg P(a) \vee Q(b) \\
C_6 &= \neg P(b)
\end{aligned}
$$

Let the ordering on ground atoms be given by $P(a) \succ P(b) \succ Q(a) \succ Q(b)$.

(i) Order the clauses in N according to the associated clause ordering $\succ_C$.

(ii) Compute the candidate interpretation $I_N^{\succ}$. Which clauses are productive, what do they produce, which clause is the minimal counterexample (if it exists)?

Submit your solution in lecture hall E1.3, Room 001 during the lecture on December 4. Please write the time of your tutorial group (Mon or Tue) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.