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Tutorials for "Automated Reasoning"
Exercise sheet 3

Exercise 3.1: *(2+4 P)*
The sudoku puzzle presented in the first lecture

	1	2	3	4	5	6	7	8	9
1								1	
2	**4**								
3		2							
4					5		4		7
5			8				3		
6			1		9				
7	3			4			2		
8		5		1					
9				8		6			

has a unique solution. If we replace the 4 in column 1, row 2, by some other digit, this need no longer hold.

(1) Use a SAT solver to find out for which values in column 1, row 2, the puzzle has no solution.

(2) Describe a set of propositional clauses that is satisfiable if and only if a sudoku puzzle has more than one solution. Use it to find out for which values in column 1, row 2, the puzzle has more than one solution.

Hint: The perl script at

`http://rg1-teaching.mpi-inf.mpg.de/autrea-ws19/gensud`

produces an encoding of the sudoku above in DIMACS CNF format, which is accepted by most SAT solvers.

Exercise 3.2: (*5 P*)
Prove part (d) of Theorem 1.10: If $\succ$ is a strict total ordering, then its multiset extension $\succ_{\mathrm{mul}}$ is a strict total ordering.

Exercise 3.3: (*4 P*)
A partial Π-valuation $\mathcal{A}$ under which all clauses of a clause set N are true is called a partial Π-model of N.

Do the following clause sets over $\Pi = \{P, Q, R\}$ have partial Π-models that are not total Π-models (that is, models in the sense of Sec. 2.3)? If yes, give such a partial Π-model.

(1)
$$\begin{array}{rcrcr} P & & & \vee & R \\ \neg P & \vee & Q & \vee & \neg R \\ & & \neg Q & \vee & \neg R \end{array}$$

(2)
$$\begin{array}{rcrcr} P & & & & \\ \neg P & \vee & Q & & \\ & & \neg Q & \vee & R \\ \neg P & \vee & \neg Q & \vee & \neg R \end{array}$$

(3)
$$\begin{array}{rcrcr} P & & & & \\ \neg P & \vee & Q & & \\ \neg P & \vee & \neg Q & \vee & \neg R \end{array}$$

(4)
$$\begin{array}{rcrcr} \neg P & \vee & Q & & \\ & & \neg Q & \vee & R \\ P & & & \vee & \neg R \end{array}$$

Exercise 3.4: (*5 P*)
For any propositional formula F let $\mathit{negvar}(F)$ be the formula obtained from F by replacing every propositional variable by its negation. (E. g., $\mathit{negvar}(P \vee (\neg Q \rightarrow (\neg P \wedge \top))) = \neg P \vee (\neg\neg Q \rightarrow (\neg\neg P \wedge \top))$.)

Prove or refute: If a formula F is satisfiable, then $\mathit{negvar}(F)$ is satisfiable. (It is suficient if you consider the boolean connectives $\neg$ and $\wedge$; the others are handled analogously.)

Submit your solution in lecture hall E1.3, Room 001 during the lecture on November 13. Please write the time of your tutorial group (Mon or Tue) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.