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Tutorials for “Automated Reasoning”
Exercise sheet 11

Exercise 11.1: (5 P)

Let (A, \rightarrow) be a reduction system such that for every $a, b, c \in A$, whenever $b \leftarrow a \rightarrow c$ then $b = c$ or there is a $d \in A$ such that $b \rightarrow d \leftarrow c$.

Show that if an element $a \in A$ has a normal form, then there is no infinite reduction sequence starting from a . (Hint: You have to strengthen the statement in order to get a property that can be proved by induction.)

Exercise 11.2: (5 P)

Compute all critical pairs for each of the following term rewrite systems. (Omit the trivial critical pairs obtained by overlapping a rule with itself at the position ε .)

- (a) $\{ f(g(f(x))) \rightarrow x, f(g(x)) \rightarrow g(f(x)) \}$
- (b) $\{ f(x, x) \rightarrow b, f(x, g(x)) \rightarrow c \}$
- (c) $\{ f(g(x)) \rightarrow x, f(c) \rightarrow c \}$
- (d) $\{ f(f(x, y), z) \rightarrow f(x, f(y, z)), f(x, 1) \rightarrow x \}$
- (e) $\{ f(f(x, y), z) \rightarrow f(x, f(y, z)), f(1, x) \rightarrow x \}$

Which systems are locally confluent?

Exercise 11.3: (4 P)

Prove Lemma 4.18: The relation \sqsupset is compatible with Σ -operations, if and only if $s \sqsupset s'$ implies $t[s]_p \sqsupset t[s']_p$ for all $s, s', t \in T_\Sigma(X)$ and $p \in \text{pos}(t)$.

Exercise 11.4: (3 P)

Let $\Sigma = (\{f/1, g/2, h/1, b/0, c/0\}, \emptyset)$ and let

$$t_1 = g(h(x), h(c)),$$

$$t_2 = g(x, h(h(x))),$$

$$t_3 = h(g(x, b)),$$

$$t_4 = f(g(h(h(x)), y)).$$

Determine for each $1 \leq i < j \leq 4$ whether t_i and t_j are incomparable or comparable (and if so, which term is larger) with respect to a polynomial ordering over $\{n \in \mathbb{N} \mid n \geq 1\}$ with $P_f(X_1) = X_1 + 1$, $P_g(X_1, X_2) = X_1 + X_2$, $P_h(X_1) = 2X_1$, $P_b = 1$ and $P_c = 3$.

Exercise 11.5: (3 P)

Find a polynomial ordering \succ over $\{n \in \mathbb{N} \mid n \geq 1\}$ with linear polynomials such that $g(x) \succ x$, $h(x) \succ g(x)$, and $f(g(x)) \succ g(h(f(x)))$.

Challenge Problem: (6 Bonus Points)

Find a signature Σ containing at least one constant symbol, a set E of Σ -equations, and two terms $s, t \in T_\Sigma(X)$ such that

$$T_\Sigma(\{x_1\})/E \models \forall \vec{x}(s \approx t),$$

but

$$T_\Sigma(\{x_1, x_2\})/E \not\models \forall \vec{x}(s \approx t)$$

where \vec{x} consists of all the variables occurring in s and t . (The variables in \vec{x} need not be contained in $\{x_1, x_2\}$.)

Submit your solution in lecture hall E1.3, Room 001 during the lecture on January 22. Please write the time of your tutorial group (Mon or Tue) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.