Assignment 1 *(OBDDs)* (3 + 6 + 3 = 12 points)

Let F be the propositional formula $P \wedge (Q \vee R) \wedge S$.

Part (a) Give the reduced OBDD for F w.r.t. the ordering $P < Q < R < S$.

Part (b) Find a total ordering over $\{P, Q, R, S\}$ such that the reduced OBDD for F has 6 non-leaf nodes. Give the resulting reduced OBDD.

Part (c) For how many total orderings over $\{P, Q, R, S\}$ does the reduced OBDD for F have 6 non-leaf nodes?

Assignment 2 *(Ordered Resolution)* (7 + 7 = 14 points)

In Sect. 3.13 of the lecture notes (p. 79), the inference rules for ground resolution with ordering restrictions (without selection functions) are given by

(Ground) Ordered Resolution:

$$\frac{D \vee A \qquad C \vee \neg A}{D \vee C} \quad \text{if } A \succ L \text{ for all } L \text{ in } D \text{ and } \neg A \succeq L \text{ for all } L \text{ in } C.$$

(Ground) Ordered Factorization:

$$\frac{C \vee A \vee A}{C \vee A} \quad \text{if } A \succeq L \text{ for all } L \text{ in } C.$$

This calculus is sound and refutationally complete for sets of ground clauses.

Suppose that we replace the ordering restriction for the first inference rule by "if $A \succ L$ for all L in D and $A \succeq L$ for all L in C."

Part (a) Is the calculus with this modification still sound? If yes, give a short explanation; if no, give a counter example.

Part (b) Is the calculus with this modification still refutationally complete? If yes, give a short explanation; if no, give a counter example.

Assignment 3 *(E-Algebras)* (12 points)

Let $\Sigma = (\Omega, \emptyset)$ be a first-order signature with $\Omega = \{f/1,\ b/0,\ c/0,\ d/0\}$. Let E be the set of Σ-equations

$$\{\, \forall x\, (f(x) \approx b),\ \ c \approx d \,\},$$

let $X = \{x, y, z\}$ be a set of variables. For any $t \in \mathrm{T}_\Sigma(X)$ let $[t]$ denote the congruence class of t w.r.t. E. Let $\mathcal{T} = \mathrm{T}_\Sigma(X)/E$, let $U_\mathcal{T}$ be the universe of $\mathcal{T}$, and let $\beta : X \to U_\mathcal{T}$ be the assignment that maps every variable to $[c]$. Decide for each of the following statements whether they are true or false:

(1) $[z]$ is a finite set of Σ-terms.

(2) $[f(z)]$ is a finite set of Σ-terms.

(3) $[c]$ is a set of ground Σ-terms.

(4) $[f(c)]$ is a set of ground Σ-terms.

(5) $U_\mathcal{T}$ is finite.

(6) $[b] \in U_\mathcal{T}$.

(7) $\{x, y\} \in U_\mathcal{T}$.

(8) $\mathcal{T}(\beta)(\forall z\, (z \approx f(x))) = 1$.

(Note on grading: A yes/no answer is sufficient; you do not have to give any explanations. However, you need at least five correct answers to get any points for assignment 3. Missing answers count like false answers.)

Assignment 4 *(Term Orderings)* (7 + 7 = 14 points)

Let $\Sigma = (\Omega, \Pi)$ be a finite signature, let $\succ$ be a strict partial ordering on Ω, let $s, t \in \mathrm{T}_\Sigma(X)$.

Part (a) Prove: If s contains a subterm $s' = f(s_1, \dots, s_n)$ such that $\mathrm{var}(s') \supseteq \mathrm{var}(t)$ and $f \succ g$ for all function symbols g occurring in t, then $s \succ_{\mathrm{lpo}} t$.

Part (b) Refute: If s contains a subterm $s' = f(s_1, \dots, s_n)$ such that $\mathrm{var}(s) \supseteq \mathrm{var}(t)$ and $f \succ g$ for all function symbols g occurring in t, then $s \succ_{\mathrm{lpo}} t$.

Assignment 5 *(Critical Pairs, Termination)* (8 + 6 = 14 points)

Let E be the following set of equations over $\Sigma = (\{f/1,\ g/2,\ h/2,\ b/0,\ c/0\}, \emptyset)$.

$$f(f(x)) \approx g(b, x) \qquad (1)$$
$$h(f(y), y') \approx f(h(y, y')) \qquad (2)$$
$$g(h(z, z), c) \approx h(z, b) \qquad (3)$$

Part (a) Suppose that the three equations in E are turned into rewrite rules by orienting them from left to right. Give all (non-trivial) critical pairs between the resulting three rules.

Part (b) It is possible to orient the equations in E using an appropriate KBO, so that there are no critical pairs between the resulting rules. Give the weights and precedence of the KBO; explain how the equations are oriented.

Assignment 6 *(Dependency Pairs)* (3 + 8 + 3 = 14 points)

Part (a) Let $\Sigma = (\Omega, \emptyset)$ with $\Omega = \{f/2,\ g/2,\ h/1,\ k/1,\ b/0\}$. Compute the dependency pairs of the following rewrite system R over Σ:

$$f(x, y) \to g(k(y), y) \qquad (1)$$
$$g(h(x), y) \to f(x, y) \qquad (2)$$
$$g(x, h(y)) \to h(f(x, y)) \qquad (3)$$
$$k(x) \to x \qquad (4)$$
$$k(h(x)) \to k(x) \qquad (5)$$

Part (b) Compute the approximated dependency graph for R (using cap and ren) and use the subterm criterion to reduce the graph as much as possible. If a graph is modified, depict both the original and the modified graph and indicate the strongly connected components in the graphs.

Part (c) There is one cycle K in the approximated dependency graph for R that cannot be eliminated using the subterm criterion. In fact, there is a K-minimal infinite rewrite sequence for this cycle. How does it look like?