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Tutorials for "Automated Reasoning"
Exercise sheet 5

Exercise 5.1: (*4 P*)
Prove Case 2 of Thm. 2.21 (i. e.: v and v' are non-leaf nodes labelled by the same propositional variable).

Exercise 5.2: (*4 P*)
Prove Prop. 3.2: For any Σ-formula F, $\mathcal{A}(\beta)(F\sigma) = \mathcal{A}(\beta \circ \sigma)(F)$.
(It is sufficient if you prove the property for atomic formulas $P(s_1, \dots, s_n)$, disjunctions $F \vee G$, and universally quantified formulas $\forall x\, F$; the other cases are proved similarly.)

Exercise 5.3: (*4 P*)
Transform the first-order formula

$$F \;=\; \forall x\, \exists y\, \exists z \Big(\big(P(x) \wedge R(x,y)\big) \vee \neg \forall w\, Q(z,w) \Big)$$

into clause normal form using the improved algorithm from Section 3.6. (There are no subformulas in F for which one should introduce a definition.)

Exercise 5.4: (*5 P*)
Let $\Sigma = (\Omega, \Pi)$ with $\Omega = \{b/0,\ c/0,\ d/0\}$ and $\Pi = \{P/1,\ Q/0\}$. Are the following statements correct? Give a brief explanation.

(1) The formula $\forall x\, P(x)$ has infinitely many Σ-models.

(2) The formula $P(b) \wedge P(c) \wedge \neg P(d)$ has a Σ-model with a two-element universe.

(3) Every Σ-model of $P(b) \wedge P(c) \wedge P(d)$ is a model of $\forall x\, P(x)$.

(4) Every Herbrand model over Σ of $P(b) \wedge P(c) \wedge P(d)$ is a model of $\forall x\, P(x)$.

(5) The formula $P(b) \wedge P(c) \wedge \neg P(d)$ has exactly two Herbrand models over Σ.

Exercise 5.5: (*2+2 P*)
Let N be the following set of ground clauses:

$$\{\, P \vee Q,\ P \vee \neg Q,\ \neg P \vee Q,\ \neg P \vee \neg Q \,\}$$

(i) Show that $N \vdash_{Res} \bot$, that is, derive $\bot$ from N using the Resolution and the Positive Factorization rule.

(ii) Why is it impossible to derive the empty clause from these clauses without using factorization?

Submit your solution in lecture hall E1.3, Room 003 during the lecture on November 30. Please write your name and the time of your tutorial group (Tue 8–10, Wed 8–10, or Wed 14–16) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.