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**Uwe Waldmann** **November 16, 2017**

**Tutorials for "Automated Reasoning"**
**Exercise sheet 4**

**Exercise 4.1:** (*4 P*)
A friend asks you to proofread his bachelor thesis. On page 20 of the thesis, your friend writes the following:

> **Definition 11.** Let $N$ be a set of propositional formulas. The set $poscomb(N)$ of positive combinations of formulas in $N$ is defined inductively by
> (1) $N \subseteq poscomb(N)$;
> (2) if $F, F' \in poscomb(N)$, then $F \vee F' \in poscomb(N)$; and
> (3) if $F, F' \in poscomb(N)$, then $F \wedge F' \in poscomb(N)$.
>
> **Lemma 12.** If $N$ is a satisfiable set of formulas, then every positive combination of formulas in $N$ is satisfiable.
>
> **Proof.** The proof proceeds by induction over the formula structure. Let $G \in poscomb(N)$. If $G \in N$, then it is obviously satisfiable, since $N$ is satisfiable. Otherwise, $G$ must be a disjunction or a conjunction of formulas in $poscomb(N)$. If $G$ is a disjunction $F \vee F'$ with $F, F' \in poscomb(N)$, we know by the induction hypothesis that $F$ and $F'$ are satisfiable. So $F$ has a model. Since this is also a model of $G = F \vee F'$, the formula $G$ is satisfiable. The proof for conjunctions is analogous.

(1) Is the "proof" correct (yes/no)?

(2) If the "proof" is not correct:

  (a) Which step is incorrect?

  (b) Does the "lemma" hold? If yes, give a correct proof, otherwise give a counterexample.

**Exercise 4.2:** (*3 P*)
The "Purity deletion" rule explained in the *Inprocessing* section is subsumed by other inprocessing rules. By which one(s)? Why?

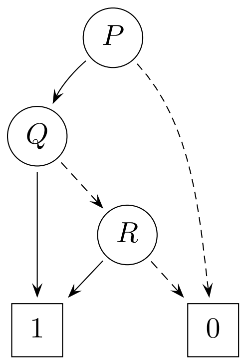
**Exercise 4.3:** (5 *P*)
Prove that the "RAT elimination" rule explained in the *Inprocessing* section is satisfiability-preserving:

$C$ is called an *asymmetric tautology* w.r.t. $N$, if its negation can be refuted by unit propagation using clauses in $N$.

We say that $C$ has the *RAT property* w.r.t. $N$, if it is an asymmetric tautology w.r.t. $N$, or if there is a literal $L$ in $C$ such that $C = C' \vee L$ and all clauses $D' \vee C'$ for $D' \vee \overline{L} \in N$ are asymmetric tautologies w.r.t. $N$.

Assume that $C$ has the RAT property w.r.t. $N$. Show that $N \cup \{C\}$ is satisfiable if and only if $N$ is satisfiable.

**Exercise 4.4:** (5 *P*)
(1) Give a propositional formula $F$ that is represented by the reduced OBDD on the right.




(2) How many different reduced OBDDs over the propositional variables $\{P, Q, R\}$ have exactly one interior (non-leaf) node?

(3) Find a propositional formula $G$ over the propositional variables $\{P, Q, R\}$, such that the reduced OBDD for $G$ has three interior nodes and the reduced OBDD for $F \vee G$ has one interior node. Give the reduced OBDDs for $G$ and $F \vee G$.

**Exercise 4.5:** (5 *P*)
Let the signature $\Sigma = (\Omega, \Pi)$ be given by $\Omega = \{+/2, s/1, 0/0\}$ and $\Pi = \emptyset$, and let

$$\begin{aligned}
F_1 &= \forall x\,(x + 0 \approx x)\\
F_2 &= \forall x \forall y\,(x + s(y) \approx s(x + y))\\
F_3 &= \forall x \forall y\,(x + y \approx y + x)\\
F_4 &= \neg \forall x \forall y\,(x + y \approx y + x).
\end{aligned}$$

(1) Determine a $\Sigma$-algebra $\mathcal{A}$ with an universe of exactly two elements such that $\mathcal{A}$ is a model of $F_1$, $F_2$, $F_3$.

(2) Determine a $\Sigma$-algebra $\mathcal{A}$ with an universe of exactly two elements such that $\mathcal{A}$ is a model of $F_1$, $F_2$, $F_4$.

Submit your solution in lecture hall E1.3, Room 003 during the lecture on November 23. Please write your name and the time of your tutorial group (Tue 8–10, Wed 8–10, or Wed 14–16) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.