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Tutorials for "Automated Reasoning"
Exercise sheet 3

Exercise 3.1: (*6 P*)
The sudoku puzzle presented in the first lecture

	1	2	3	4	5	6	7	8	9
1								1	
2	**4**								
3		2							
4					5		4		7
5			8				3		
6			1		9				
7	3			4			2		
8		5		1					
9				8		6			

has a unique solution. If we replace the 4 in column 1, row 2, by some other digit, this need no longer hold. Use a SAT solver to find values in column 1, row 2, such that the puzzle has

(1) no solution,
(2) more than one solution.

Explain how you found the values. Hint: The perl script at

`http://resources.mpi-inf.mpg.de/departments/rg1/teaching/autrea-ws17/gensud`

produces an encoding of the sudoku above in DIMACS CNF format, which is accepted by most SAT solvers.

Exercise 3.2: (*7 P*)
Let N be the following set of propositional clauses:

$$
\begin{array}{rcccccccccccccc}
(1) & P & \vee & \neg Q & \vee & R & & & & & & & & & \\
(2) & P & & & & & & & \vee & \neg T & \vee & \neg U & \vee & V \\
(3) & P & \vee & \neg Q & & & & & \vee & T & \vee & U & \vee & \neg V \\
(4) & \neg P & \vee & Q & & & & & & & & & & \\
(5) & & & & & R & & & \vee & T & & & & \\
(6) & & & & & R & & & & & \vee & \neg U & & \\
(7) & \neg P & & & & & \vee & S & & & \vee & \neg U & \vee & \neg V \\
(8) & & & & & \neg R & \vee & S & & & & & & \\
(9) & & & & & \neg R & & & \vee & T & & & \vee & V \\
(10) & & & & & & & \neg S & \vee & T & \vee & U & \vee & \neg V \\
(11) & & & & & & & & & \neg T & \vee & U & & \\
(12) & & & & & & & \neg S & \vee & \neg T & \vee & \neg U & \vee & V \\
(13) & & & & & & & & & & & \neg U & \vee & \neg V
\end{array}
$$

Use the CDCL procedure to check whether N is satisfiable or not; if it is satisfiable, give a model. Use the CDCL inference rules with a reasonable strategy (i.e., use *Fail* or *Backjump* if possible, otherwise use *Unit Propagate* if possible, otherwise use *Decide*). If you use the *Decide* rule, use the largest undefined positive literal according to the ordering $P > Q > R > S > T > U > V$. If you use the *Backjump* rule, determine a suitable backjump clause using the 1UIP method and use the best possible successor state for that backjump clause.

Exercise 3.3: (*5 P*)
For any propositional formula F let *negvar*(F) be the formula obtained from F by replacing every propositional variable by its negation. (E. g., $\mathit{negvar}(P \vee (\neg Q \rightarrow (\neg P \wedge \top))) = \neg P \vee (\neg\neg Q \rightarrow (\neg\neg P \wedge \top))$.)

Prove or refute: If a formula F is satisfiable, then *negvar*(F) is satisfiable.

Submit your solution in lecture hall E1.3, Room 003 during the lecture on November 16. Please write your name and the time of your tutorial group (Tue 8–10, Wed 8–10, or Wed 14–16) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.