Uwe Waldmann **January 18, 2018**

Tutorials for "Automated Reasoning"
Exercise sheet 12

Exercise 12.1: (*5 P*)
Prove Thm. 4.30: If the precedence $\succ$ is total, then the lexicographic path ordering $\succ_{\text{lpo}}$ is total on ground terms, i.e., for all $s, t \in \mathrm{T}_\Sigma(\emptyset)$: $s \succ_{\text{lpo}} t \vee t \succ_{\text{lpo}} s \vee s = t$.

Exercise 12.2: (*6 P*)
Let $\Sigma = (\{f/1,\ g/2,\ h/1,\ b/0,\ c/0\}, \emptyset)$ and let

$$\begin{aligned} t_1 &= g(h(x), h(c)), \\ t_2 &= g(x, h(h(x))), \\ t_3 &= h(g(x, b)), \\ t_4 &= f(g(x, y)). \end{aligned}$$

Determine for each $1 \leq i < j \leq 4$ whether t_i and t_j are uncomparable or comparable (and if so, which term is larger) with respect to

(a) a lexicographic path ordering with precedence $f > g > h > b > c$,

(b) a Knuth-Bendix-ordering with precedence $h > f > g > b > c$, where h has weight 0 and all other symbols have weight 1,

(c) a polynomial ordering over $\{\, n \in \mathbb{N} \mid n \geq 1 \,\}$ with $P_f(X_1) = X_1 + 1$, $P_g(X_1, X_2) = X_1 + X_2$, $P_h(X_1) = 2X_1$, $P_b = 1$ and $P_c = 3$.

Exercise 12.3: *(1+1+2 P)*

- Find a polynomial ordering $\succ$ over $\{\, n \in \mathbb{N} \mid n \geq 1 \,\}$ with linear polynomials such that $g(x) \succ x$, $h(x) \succ g(x)$, and $f(g(x)) \succ g(h(f(x)))$.
- Find a lexicographic path ordering $\succ$ such that $h(h(x)) \succ f(x)$ and $f(g(h(x), y)) \succ h(g(x, f(y)))$.
- Find a Knuth-Bendix-ordering $\succ$ such that the set of clauses

$$P(f(x,y),y) \vee P(g(y),g(x)) \qquad (1)$$
$$\neg P(f(b,y),y) \vee \neg Q(y,y) \qquad (2)$$
$$Q(g(x),g(y)) \vee \neg Q(x,h(y)) \qquad (3)$$

  is saturated under $Res^{\succ}_{sel}$, where *sel* does not select any literals.

Exercise 12.4: *(5 P)*
Apply the Knuth-Bendix procedure to the set of equations

$$f(f(x)) \approx g(x) \qquad (1)$$
$$f(b) \approx c \qquad (2)$$

and transform it into a finite convergent term rewrite system; use the Knuth-Bendix ordering with weight 1 for all function symbols and variables and the precedence $g > f > b > c$.

Submit your solution in lecture hall E1.3, Room 003 during the lecture on January 25. Please write your name and the time of your tutorial group (Tue 8–10, Wed 8–10, or Wed 14–16) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.