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Tutorials for "Automated Reasoning"
Exercise sheet 10

Exercise 10.1: *(3+5 P)*

(a) Show that the compactness theorem (Thm. 3.39) holds also for first-order logic with equality. (You may use all results proved in the lecture so far.)

(b) Use the compactness theorem for first-order logic with equality to prove the following statement: Let F be a first-order formula with equality. If, for every natural number n, F has a model whose universe has at least n elements, then F has a model with an infinite universe.

Exercise 10.2: *(4 P)*
Let $E = \{\, f(g(x)) \approx g(f(x)) \,\}$. Give a derivation for $E \vdash f(f(g(g(y)))) \approx g(g(f(f(y))))$.

Exercise 10.3: *(4 P)*
Consider the signature $\Sigma = (\{f/1, b/0, c/0\}, \emptyset)$ and the set of (implicitly universally quantified) equations $E = \{\, f(f(x)) \approx x \,\}$. How many elements does the universe of $\mathrm{T}_\Sigma(\emptyset)/E$ have? How do they look like? How is $f_{\mathrm{T}_\Sigma(\emptyset)/E}$ defined on the universe?

Exercise 10.4: *(5 P)*
Let $\Sigma = (\{f/2, b/0, c/0, d/0\}, \emptyset)$; let $E = \{\forall x\,(f(x,x) \approx b),\ c \approx d\}$; let $X = \{x, y, z\}$ be a set of variables. For any $t \in \mathrm{T}_\Sigma(X)$ let $[t]$ denote the congruence class of t w.r.t. E. Let $\mathcal{T} = \mathrm{T}_\Sigma(X)/E$ and let $\beta : X \to \mathcal{T}$ be the assignment that maps every variable to $[c]$.

State for each of the following "items" whether it is (a) a term, (b) a set of terms, (c) a set of ground terms, (d) a congruence class w.r.t. E, (e) a formula, (f) a truth value, or (g) non-sensical. (More than one answer may be correct.)

(1)	$[c]$	(6)	$f([c],[d])$
(2)	$[b]$	(7)	$f_{\mathcal{T}}(\{c,d\},\{c,d\})$
(3)	$\{d\}$	(8)	$b \approx c$
(4)	$f(c,d)$	(9)	$\mathcal{T}(\beta)(\forall x, y\ (f(x,x) \approx f(y,y)))$
(5)	$[f(c,d)]$	(10)	$f_{\mathcal{T}}(\mathcal{T}(\beta)(x), \mathcal{T}(\beta)(y))$

Submit your solution in lecture hall E1.3, Room 003 during the lecture on January 11. Please write your name and the time of your tutorial group (Tue 8–10, Wed 8–10, or Wed 14–16) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.