max planck institut informatik

Universität des Saarlandes
FR Informatik

Uwe Waldmann January 4, 2016

Tutorials for "Automated Reasoning"
Exercise sheet 9

Exercise 9.1: *(3+3 P)*
Let $\Sigma = (\Omega, \Pi)$ be a signature with $\Omega = \{b/0, f/1\}$ and $\Pi = \{P/1\}$. Suppose that the atom ordering $\succ$ is defined on ground atoms in such a way that $P(f^n(b)) \succ P(f^m(b))$ if and only if $n > m \geq 0$. Let N be the following set of clauses:

$$P(f(f(b))$$
$$\neg P(x) \vee P(f(x))$$

(a) Sketch how the set $G_\Sigma(N)$ of all ground instances of clauses in N looks like. How is it ordered with respect to the clause ordering $\succ_C$?

(b) Construct the candidate interpretation $I^{\succ}_{G_\Sigma(N)}$ of the set of all ground instances of clauses in N.

Exercise 9.2: *(3+3 P)*
Suppose that the atom ordering $\succ$ compares ground atoms by comparing lexicographically first the predicate symbols ($P > Q > R$), then the size of the first argument, and then the size of the second argument (if present). If at least one of the two atoms to be compared is non-ground, $\succ$ compares only the predicate symbols.

Let N be the following set of clauses:

$$P(f(x), x) \vee R(b, b) \qquad (1)$$
$$\neg P(b, x) \vee \neg P(x, b) \vee Q(x) \qquad (2)$$
$$Q(f(b)) \vee \neg Q(b) \vee R(f(x), b) \qquad (3)$$
$$Q(b) \vee \neg R(f(x), f(x)) \qquad (4)$$
$$\neg Q(x) \vee R(x, x) \qquad (5)$$

(a) Which literals are maximal in the clauses of N?

(b) Define a selection function sel such that N is saturated under $Res^{\succ}_{sel}$.

Exercise 9.3: (*3+3 P*)
Let N be the following set of ground clauses:

$$\neg P_3 \vee P_1 \vee P_1 \qquad (1)$$
$$\neg P_2 \vee P_1 \qquad (2)$$
$$P_4 \vee P_4 \qquad (3)$$
$$P_3 \vee \neg P_2 \qquad (4)$$
$$P_4 \vee P_3 \qquad (5)$$

(a) Find a total atom ordering $\succ$ such that both clause (2) and (5) are redundant w.r.t. N.

(b) Prove that there is no atom ordering such that clause (4) is redundant w.r.t. N.

Challenge Problem: (*4 Bonus Points*)
Prove that it is undecidable whether a clause C is redundant w. r. t. a set of clauses N. (You may use the fact that the satisfiability of a set of first-order clauses is undecidable.)

Submit your solution in lecture hall E1.3, Room 003 during the lecture on January 11. Please write your name and the time of your tutorial group (Mo 8–10 or Mo 12–14) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.