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**Uwe Waldmann** **December 7, 2015**

**Tutorials for "Automated Reasoning"**
**Exercise sheet 7**

**Exercise 7.1:** *(4+2 P)*
Let $N$ be the following set of ground clauses:

$$\{\, P \vee Q,\ P \vee \neg Q,\ \neg P \vee Q,\ \neg P \vee \neg Q \,\}$$

(i) Show that $N \vdash_{Res} \bot$, that is, derive $\bot$ from $N$ using the Resolution and the Positive Factorization rule.

(ii) Why is it impossible to derive the empty clause from these clauses without using factorization?

**Exercise 7.2:** *(4+4 P)*
Let $N$ be the set containing the following ground clauses:

$$\begin{aligned}
C_1 &= P(a) \vee \neg Q(a) \vee \neg Q(b) \\
C_2 &= P(a) \vee P(a) \vee Q(b) \\
C_3 &= P(b) \vee Q(a) \vee Q(b) \\
C_4 &= Q(a) \vee Q(b) \vee Q(b) \\
C_5 &= \neg P(a) \vee Q(b) \\
C_6 &= \neg P(b)
\end{aligned}$$

Let the ordering on ground atoms be given by $P(a) \succ P(b) \succ Q(a) \succ Q(b)$.

(i) Order the clauses in $N$ according to the associated clause ordering $\succ_C$.

(ii) Compute the candidate interpretation $I_N^{\succ}$. Which clauses are productive, what do they produce, which clause is the minimal counterexample (if it exists)?

**Exercise 7.3:** *(4 P)*
Find a finite set $N$ of ground clauses such that no clause in $N$ is a tautology and such that $Res^*(N)$ is infinite.

**Challenge Problem:** (*4 Bonus Points*)
Let $\Sigma = (\Omega, \Pi)$ be a signature such that $\Omega$ contains at least one constant symbol. A $\Sigma$-algebra $\mathcal{A}$ is called *term-generated*, if every $a \in U_{\mathcal{A}}$ is term-generated. Prove that a closed prenex formula without existential quantifiers has a model if and only if it has a term-generated model.

Submit your solution in lecture hall E1.3, Room 003 during the lecture on December 14. Please write your name and the time of your tutorial group (Mo 8–10 or Mo 12–14) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.