max planck institut informatik

Universität des Saarlandes FR Informatik



Uwe Waldmann

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## Tutorials for "Automated Reasoning" Exercise sheet 5

## **Exercise 5.1:** (4 P)

When we implement CDCL using the "two watched literals" approach, we initially select two "watched" literals in every clause. If a previously undefined literal L becomes false, we check every clause C in which this literal is watched and watch another literal (that is true or undefined) in this clause if possible. What should we do if we notice that the other watched literal of C has already become true in an earlier step (and if all other literals in the clause have become false in the meantime, so that there is no further true or undefined literal in C? Is there a solution that does not require explicit update operations upon backjumping? Explain.

## **Exercise 5.2:** (5 P)

(1) Give a propositional formula F that is represented by the reduced OBDD on the right.

(2) How many different reduced OBDDs over the propositional variables  $\{P, Q, R\}$  have exactly one interior (non-leaf) node?

(3) Find a propositional formula G over the propositional variables  $\{P, Q, R\}$ , such that the reduced OBDD for G has three interior nodes and the reduced OBDD for  $F \vee G$  has one interior node. Give the reduced OBDDs for G and  $F \vee G$ .



**Exercise 5.3:** (5 P) Let the signature  $\Sigma = (\Omega, \Pi)$  be given by  $\Omega = \{+/2, s/1, 0/0\}$  and  $\Pi = \emptyset$ , and let

$$\begin{array}{rcl} F_1 &=& \forall x \, (x+0 \approx x) \\ F_2 &=& \forall x \forall y \, (x+s(y) \approx s(x+y)) \\ F_3 &=& \forall x \forall y \, (x+y \approx y+x) \\ F_4 &=& \neg \forall x \forall y \, (x+y \approx y+x). \end{array}$$

- (1) Determine a  $\Sigma$ -algebra  $\mathcal{A}$  with an universe of exactly two elements such that  $\mathcal{A}$  is a model of  $F_1$ ,  $F_2$ ,  $F_3$ .
- (2) Determine a  $\Sigma$ -algebra  $\mathcal{A}$  with an universe of exactly two elements such that  $\mathcal{A}$  is a model of  $F_1$ ,  $F_2$ ,  $F_4$ .

## **Exercise 5.4:** (4 P)

Prove Prop. 3.2: For any  $\Sigma$ -formula F,  $\mathcal{A}(\beta)(F\sigma) = \mathcal{A}(\beta \circ \sigma)(F)$ . (It is sufficient if you prove the property for atomic formulas  $P(s_1, \ldots, s_n)$ , disjunctions  $F \lor G$ , and universally quantified formulas  $\forall x F$ ; the other cases are proved similarly.)

Submit your solution in lecture hall E1.3, Room 003 during the lecture on November 30. Please write your name and the time of your tutorial group (Mo 8–10 or Mo 12–14) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.