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Tutorials for "Automated Reasoning"
Exercise sheet 4

Exercise 4.1: (*8 P*)
Let N be the following set of propositional clauses:

$$
\begin{array}{lllllllllllllllllll}
(1) & P & \vee & \neg Q & \vee & R & & & & & & & & & & & & \\
(2) & P & & & & & & & & \vee & \neg T & \vee & \neg U & \vee & V & & & \\
(3) & P & \vee & \neg Q & & & & & & \vee & T & \vee & U & \vee & \neg V & & & \\
(4) & \neg P & \vee & Q & & & & & & & & & & & & & & \\
(5) & & & & & R & & & & \vee & T & & & & & & & \\
(6) & & & & & R & & & & & & \vee & \neg U & & & & & \\
(7) & \neg P & & & & & \vee & S & & & & \vee & \neg U & \vee & \neg V & & & \\
(8) & & & & & \neg R & \vee & S & & & & & & & & & & \\
(9) & & & & & \neg R & & & & \vee & T & & & \vee & V & & & \\
(10) & & & & & & & \neg S & \vee & T & \vee & U & \vee & \neg V & & & & \\
(11) & & & & & & & & & \neg T & \vee & U & & & & & & \\
(12) & & & & & & & \neg S & \vee & \neg T & \vee & \neg U & \vee & V & & & & \\
(13) & & & & & & & & & & & \neg U & \vee & \neg V & & & &
\end{array}
$$

Use the CDCL procedure to check whether N is satisfiable or not; if it is satisfiable, give a model. Use the CDCL inference rule with a reasonable strategy. If you use the *Decide* rule, use the largest undefined positive literal according to the ordering $P > Q > R > S > T > U > V$. If you use the *Backjump* rule, determine a suitable backjump clause using the 1UIP method and use the best possible successor state for that backjump clause.

Exercise 4.2: (*6 P*)
Prove that the "Literal Elimination" rule explained in the *Preprocessing* section is satisfiability-preserving: Let N be a set of clauses in which the propositional variable P does not occur. For $1 \leq i \leq m$ let $C_i \vee P$ be a clause in which P occurs positively; for $1 \leq j \leq n$ let

$D_j \vee \neg P$ be a clause in which P occurs negatively. Then

$$N \cup \{\, C_i \vee P \mid 1 \leq i \leq m \,\} \cup \{\, D_j \vee \neg P \mid 1 \leq j \leq n \,\}$$

is satisfiable if and only if

$$N \cup \{\, C_i \vee D_j \mid 1 \leq i \leq m,\, 1 \leq j \leq n \,\}$$

is satisfiable.

Exercise 4.3: (*4 P*)
Give reduced OBDDs for the following four formulas:

(1) $\neg Q$

(2) $\neg(P \leftrightarrow Q)$

(3) $(P \wedge Q) \vee (Q \wedge R) \vee (R \wedge P)$

(4) $Q \vee \neg(P \wedge Q)$

Challenge Problem: (*6 Bonus Points*)
Show that the following property holds: If the inference rules of the CDCL procedure are applied with a reasonable strategy, i. e.,

- *Unit Propagate* is used only if neither *Fail* nor *Backjump* are applicable;
- *Decide* is used only if neither *Unit Propagate* nor *Fail* nor *Backjump* are applicable;
- if *Backjump* is used to go to $M'\ L' \parallel N$, with a backjump clause $C \vee L'$, then M' is the shortest list of literals that satisfies the conditions of the *Backjump* rule for $C \vee L'$;
- a clause is learned if and only if it is a backjump clause;

then the resolution process on page 30 of the lecture notes does never produce the empty clause.

Submit your solution in lecture hall E1.3, Room 003 during the lecture on November 23. Please write your name and the time of your tutorial group (Mo 8–10 or Mo 12–14) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.