max planck institut
informatik

Universität
des
Saarlandes
FR Informatik

Uwe Waldmann

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Tutorials for “Automated Reasoning”
Exercise sheet 3

Exercise 3.1: (*5 P*)
The sudoku puzzle presented in the first lecture

	1	2	3	4	5	6	7	8	9
1								1	
2	**4**								
3		2							
4					5		4		7
5			8				3		
6			1		9				
7	3			4			2		
8		5		1					
9				8		6			

has a unique solution. If we replace the 4 in column 1, row 2, by some other digit, this need no longer hold. Use a SAT solver to find values in column 1, row 2, such that the puzzle has

(1) no solution,
(2) more than one solution.

Explain how you found the values. Hint: The perl script at
`http://resources.mpi-inf.mpg.de/departments/rg1/teaching/autrea-ws15/gensud`
produces an encoding of the sudoku above in DIMACS CNF format, which is accepted by most SAT solvers.

Exercise 3.2: (*4 P*)
Let F be the formula $(Q \rightarrow P) \rightarrow (\neg P \wedge Q \wedge R)$.

(1) Convert F into an equivalent CNF formula as described in Prop. 2.11.

(2) Replace the subformulas $Q \rightarrow P$ and $\neg P \wedge Q \wedge R$ by new variables, add the polarity-dependent definitions for the new variables and convert again into a CNF formula.

Exercise 3.3: (*5 P*)
Prove Proposition 2.13: Let $\mathcal{A}$ be a valuation, let F and G be formulas, and let $H = H[F]_p$ be a formula in which F occurs as a subformula at position p.

If $\mathrm{pol}(H, p) = 1$ and $\mathcal{A}(F) \leq \mathcal{A}(G)$, then $\mathcal{A}(H[F]_p) \leq \mathcal{A}(H[G]_p)$.

If $\mathrm{pol}(H, p) = -1$ and $\mathcal{A}(F) \geq \mathcal{A}(G)$, then $\mathcal{A}(H[F]_p) \leq \mathcal{A}(H[G]_p)$.

(It is sufficient if you consider the boolean connectives $\wedge$ and $\neg$; the other cases are proved analogously.)

Exercise 3.4: (*4 P*)
A friend asks you to proofread his bachelor thesis. On page 20 of the thesis, your friend writes the following:

> **Definition 11.** Let N be a set of formulas. The set $\mathit{poscomb}(N)$ of positive combinations of formulas in N is defined inductively by
> (1) $N \subseteq \mathit{poscomb}(N)$;
> (2) if $F, F' \in \mathit{poscomb}(N)$, then $F \vee F' \in \mathit{poscomb}(N)$; and
> (3) if $F, F' \in \mathit{poscomb}(N)$, then $F \wedge F' \in \mathit{poscomb}(N)$.
>
> **Lemma 12.** If N is a satisfiable set of formulas, then every positive combination of formulas in N is satisfiable.
>
> **Proof.** The proof proceeds by induction over the formula structure. Let $G \in \mathit{poscomb}(N)$. If $G \in N$, then it is obviously satisfiable, since N is satisfiable. Otherwise, G must be a disjunction or a conjunction of formulas in $\mathit{poscomb}(N)$. If G is a disjunction $F \vee F'$ with $F, F' \in \mathit{poscomb}(N)$, we know by the induction hypothesis that F and F' are satisfiable. So F has a model. Since this is also a model of $G = F \vee F'$, the formula G is satisfiable. The proof for conjunctions is analogous.

(1) Is the "proof" correct (yes/no)?

(2) If the "proof" is not correct:

   (a) Which step is incorrect?

   (b) Does the "theorem" hold? If yes, give a correct proof, otherwise give a counterexample.

Submit your solution in lecture hall E1.3, Room 003 during the lecture on November 16. Please write your name and the time of your tutorial group (Mo 8–10 or Mo 12–14) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.