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Tutorials for “Automated Reasoning”
Exercise sheet 2

Exercise 2.1: (5 P)

Let (M, \succ) be an ordering and $a, b \in M$. We say that b is a *successor* of a , if $b \succ a$ and if there exists no $c \in M$ with $b \succ c$ and $c \succ a$.

Prove the following statements.

- (1) Prove: If (M, \succ) is well-founded, then every element of M has either a successor or it is maximal.
- (2) Prove: If (M, \succ) is well-founded and \succ is total, then the successor is unique.
- (3) If $|M| \geq 2$ and \succ is well-founded and total, then there are multisets over M which are neither minimal nor successors of any other multiset with respect to \succ_{mul} . Why?

Exercise 2.2: (5 P)

Determine which of the following formulas are valid/satisfiable/unsatisfiable:

- (1) $(P \vee Q) \rightarrow P$
- (2) $P \rightarrow (Q \rightarrow P)$
- (3) $Q \rightarrow \neg Q$
- (4) $Q \wedge \neg Q$
- (5) $\neg(\neg P \vee \neg\neg P)$

Exercise 2.3: (5 P)

Let F , G and H be propositional formulas. Prove or refute:

- (1) If $F \models G$ and $G \models H$, then $F \models H$.
- (2) If F is satisfiable and G is satisfiable, then $F \wedge G$ is satisfiable.
- (3) If F is satisfiable and $F \rightarrow G$ is satisfiable, then G is satisfiable.
- (4) If $F \vee G$ is valid, then F is valid or G is satisfiable.
- (5) If $F \vee G[F]_p$ is valid, then $F \vee G[\perp]_p$ is valid.

Exercise 2.4: (3 P)

Prove Prop. 2.6: If N is a set of propositional formulas, then $N \models F$ if and only if $N \cup \{\neg F\}$ is unsatisfiable. (A set of propositional formulas is unsatisfiable, if and only if for every valuation \mathcal{A} there is a formula G in the set such that $\mathcal{A} \not\models G$.)

Submit your solution in lecture hall E1.3, Room 003 during the lecture on November 9. Please write your name and the time of your tutorial group (Mo 8–10 or Mo 12–14) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.