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Tutorials for "Automated Reasoning"
Exercise sheet 12

Exercise 12.1: (*4 P*)
Compute all critical pairs for each of the following term rewrite systems. (Omit the trivial critical pairs obtained by overlapping a rule with itself at the position ε.)

(a) $\{\, f(g(f(x))) \to x,$
$\quad f(g(x)) \to g(f(x)) \,\}$

(b) $\{\, f(x,x) \to a,$
$\quad f(x,g(x)) \to b \,\}$

(c) $\{\, f(f(x,y),z) \to f(x,f(y,z)),$
$\quad f(x,1) \to x \,\}$

(d) $\{\, f(f(x,y),z) \to f(x,f(y,z)),$
$\quad f(1,x) \to x \,\}$

Which systems are locally confluent?

Exercise 12.2: (*5 P*)
Prove Thm. 4.30: If the precedence $\succ$ is total, then the lexicographic path ordering $\succ_{\text{lpo}}$ is total on ground terms, i.e., for all $s,t \in \mathrm{T}_\Sigma(\emptyset)$: $s \succ_{\text{lpo}} t \vee t \succ_{\text{lpo}} s \vee s = t$.

Exercise 12.3: (*6 P*)
Let $\Sigma = (\{f/1,\ g/2,\ h/1,\ b/0,\ c/0\}, \emptyset)$ and let

$$\begin{aligned} t_1 &= g(h(x), h(c)), \\ t_2 &= g(x, x), \\ t_3 &= h(g(x, c)), \\ t_4 &= f(g(x, y)). \end{aligned}$$

Determine for each $1 \leq i < j \leq 4$ whether t_i and t_j are uncomparable or comparable (and if so, which term is larger) with respect to

(a) a lexicographic path ordering with precedence $f > g > h > b > c$,

(b) a Knuth-Bendix-ordering with precedence $h > f > g > b > c$, where h has weight 0 and all other symbols have weight 1,

(c) a polynomial ordering over $\{\, n \in \mathbb{N} \mid n \geq 1 \,\}$ with $P_f(X_1) = X_1 + 1$, $P_g(X_1, X_2) = 2X_1 + X_2$, $P_h(X_1) = 3X_1$, $P_b = 1$ and $P_c = 3$.

Exercise 12.4: (*3 P*)

- Find a polynomial ordering $\succ$ over $\{\, n \in \mathbb{N} \mid n \geq 1 \,\}$ with linear polynomials such that $g(x) \succ x$, $h(x) \succ g(x)$, and $f(g(x)) \succ g(h(f(x)))$.
- Find a lexicographic path ordering $\succ$ such that $h(h(x)) \succ f(x)$ and $f(g(h(x), y)) \succ h(g(x, f(y)))$.
- Find a Knuth-Bendix-ordering $\succ$ such that the set of clauses

$$P(f(x, y), y) \vee P(g(y), g(x)) \qquad (1)$$
$$\neg P(f(b, y), y) \vee \neg Q(y, y) \qquad (2)$$
$$Q(g(x), g(y)) \vee \neg Q(x, h(y)) \qquad (3)$$

  is saturated under $Res^{\succ}_{sel}$, where *sel* does not select any literals.

Submit your solution in lecture hall E1.3, Room 003 during the lecture on February 1. Please write your name and the time of your tutorial group (Mo 8–10 or Mo 12–14) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.