max planck institut informatik

Universität des Saarlandes
FR Informatik

Uwe Waldmann **January 11, 2016**

Tutorials for "Automated Reasoning"
Exercise sheet 10

Exercise 10.1: (*4 P*)
Let N be a set of ground clauses, let $\succ$ be a total and well-founded atom ordering. Prove or refute: If every clause in N is redundant with respect to N, then every clause in N is a tautology.

Exercise 10.2: (*3 P*)
Derive a maximal strict tableau for the set of formulas

$$P \rightarrow (Q \rightarrow R) \qquad (1)$$
$$P \rightarrow Q \qquad (2)$$
$$P \wedge \neg R \qquad (3)$$

Exercise 10.3: (*4 P*)
Refute the following set of formulas using one of the two variants of the tableau calculus for first-order formulas:

$$\forall x\, \exists y\, P(x, y) \qquad (1)$$
$$\exists z\, \forall w\, \neg P(f(z), w) \qquad (2)$$

(If you use tableaux with free variables, use $v_1, v_2, v_3, \ldots$ as names for free variables.)

Exercise 10.4: (*4+5 P*)

(a) Show that the compactness theorem (Thm. 3.37) holds also for first-order logic with equality.

(b) Use the compactness theorem for first-order logic with equality to prove the following statement: Let F be a first-order formula with equality. If, for every natural number n, F has a model whose universe has at least n elements, then F has a model with an infinite universe.

Submit your solution in lecture hall E1.3, Room 003 during the lecture on January 18. Please write your name and the time of your tutorial group (Mo 8–10 or Mo 12–14) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.