Assignment 1 *(DPLL)* (12 points)

Let N be any set of propositional clauses such that no clause in N contains more than one positive literal. Prove: If $\varepsilon \parallel N \Rightarrow^*_{\mathrm{DPLL}} M \parallel N$, and if neither "Unit Propagate", nor "Fail", nor "Backjump" is applicable in the state $M \parallel N$, then N is satisfiable.

Assignment 2 *(First-Order Logic)* (12 points)

Let $\Sigma = (\Omega, \Pi)$ be a first-order signature with $\Omega = \{b/0,\ c/0,\ f/1\}$ and $\Pi = \{P/1\}$. Let F be the Σ-formula

$$P(f(b)) \wedge \neg P(f(c)) \wedge \forall x\, (\neg P(x) \vee P(f(x))).$$

Decide for each of the following statements whether they are true or false:

(1) There is a model $\mathcal{A}$ of F and an assignment β such that $\mathcal{A}(\beta)(b) \in P_{\mathcal{A}}$.

(2) There is a model $\mathcal{A}$ of F and an assignment β such that $\mathcal{A}(\beta)(c) \in P_{\mathcal{A}}$.

(3) There is a model $\mathcal{A}$ of F such that $P_{\mathcal{A}} = \emptyset$.

(4) There is a model $\mathcal{A}$ of F such that $f_{\mathcal{A}}(a) = a$ for every a in the universe of $\mathcal{A}$.

(5) There is a model of F whose universe has exactly one element.

(6) There is a Herbrand model of F whose universe has finitely many elements.

(Note on grading: A yes/no answer is sufficient; you do not have to give an explanation. However, you need at least four correct answers in this assignment to get any points. Missing answers count like false answers.)

Assignment 3 *(Redundancy)* (12 points)

Let Σ be a first-order signature and let $\succ$ be a well-founded ordering on ground atoms. Let N be a set of Σ-clauses and let C be a Σ-clause. Prove: If every ground instance of C is redundant w.r.t. $G_\Sigma(N \cup \{C\})$, then every ground instance of C is redundant w.r.t. $G_\Sigma(N)$.

Assignment 4 *(Ordered Resolution)* (9 + 9 = 18 points)

Let $\Sigma = (\Omega, \Pi)$ be a first-order signature with $\Omega = \{f/1,\ g/1,\ b/0\}$ and $\Pi = \{P/2,\ Q/1,\ R/1,\ S/1\}$; let N be the following set of clauses over Σ:

$$P(x, f(x)) \ \lor\ Q(x) \qquad (1)$$
$$\neg P(f(b), f(y)) \ \lor\ \neg R(g(y)) \ \lor\ S(y) \qquad (2)$$
$$\neg Q(x) \ \lor\ \neg Q(f(y)) \ \lor\ \neg S(x) \ \lor\ \neg R(g(y)) \qquad (3)$$
$$\neg Q(g(x)) \ \lor\ Q(b) \qquad (4)$$
$$Q(f(f(b))) \qquad (5)$$
$$S(b) \qquad (6)$$

Part (a)

Suppose that the atom ordering $\succ$ is an LPO with the precedence $P > Q > R > S > f > g > b$. Compute all ordered resolution inferences between the clauses (1)–(6) with respect to $\succ$. (Compute only inferences between the clauses given here, not between derived clauses. Do not compute any inferences that violate the ordering conditions of ordered resolution.)

Part (b)

If a selection function is defined appropriately, the set N is saturated under ordered resolution with selection (w.r.t. the ordering $\succ$ from Part (a)). Which literals have to be selected?

Assignment 5 *(Tableaux)* (12 points)

Use semantic tableaux to determine whether the following set of formulas over $\Sigma = (\emptyset, \{P/0, Q/0, R/0\})$ is satisfiable or not.

$$(P \land Q) \lor (R \land \neg Q) \qquad (1)$$
$$(\neg P \land R) \land (Q \lor \neg R) \qquad (2)$$

Use exactly the expansion rules given in the lecture; do not use shortcuts or normal form transformations.

Assignment 6 *(Knuth-Bendix Completion)* (14 points)

Let E be the following set of equations over $\Sigma = (\{f/1,\ g/1,\ b/0\}, \emptyset)$.

$$f(g(x)) \approx x \qquad (1)$$
$$g(g(b)) \approx g(b) \qquad (2)$$

Apply the Knuth-Bendix completion procedure to E and transform it into a finite convergent term rewrite system; use the Knuth-Bendix ordering with weight 1 for all function symbols and variables and the precedence $g > f > b$.