Assignment 1 *(Propositional Logic)* (6 + 6 = 12 points)

Part (a)

Prove or refute: If F and G are propositional formulas, such that F is satisfiable and $F \land G$ is unsatisfiable, then $F \land \neg G$ is satisfiable.

Part (b)

Prove or refute: If F and G are propositional formulas, such that both $\neg F$ and $\neg G$ are satisfiable, then $F \lor G$ is not valid.

Assignment 2 *(First-order Logic, Ordered Resolution)* (14 points)

Let $\Sigma = (\Omega, \Pi)$ be a first-order signature with $\Omega = \{b/0\}$ and $\Pi = \{P/3,\ Q/2,\ R/1\}$. Let

$$
\begin{aligned}
F_1 &= \forall x\, R(x) \\
F_2 &= \neg \exists x\, Q(x, b) \\
F_3 &= \forall x\, \forall y\, \big(R(y) \rightarrow P(b, y, x)\big) \\
F_4 &= \forall x\, \exists y\, \forall z\, \big(R(z) \land \neg Q(x, y) \land \neg Q(x, b) \land P(y, x, z)\big)
\end{aligned}
$$

Use ordered resolution to show that the formula $(F_1 \land F_2 \land F_3) \rightarrow F_4$ is valid. Use an atom ordering in which $P(\dots) \succ Q(\dots) \succ R(\dots)$; do not compute any inferences that violate the conditions of ordered resolution.

Assignment 3 *(Redundancy)* (8 + 4 = 12 points)

Let $\Sigma = (\Omega, \Pi)$ be a first-order signature with $\Omega = \{f/1, b/0\}$ and $\Pi = \{P/1, Q/1\}$. Suppose that the atom ordering $\succ$ is a lexicographic path ordering with the precedence $P > Q > f > b$. Let $N = \{C_1, C_2, C_3\}$ with

$$
\begin{aligned}
C_1 &= Q(b) \\
C_2 &= \neg Q(x) \lor Q(f(f(x))) \\
C_3 &= \neg Q(x) \lor P(f(x))
\end{aligned}
$$

Part (a)

Show that the clause $C_0 = P(x) \lor Q(x)$ is redundant with respect to N.

Part (b)

The clause C_0 is *not* redundant with respect to N, if we extend the signature Σ, for instance by adding one more constant symbol $c/0$ to Ω. Why? Give a brief explanation.

Assignment 4 *(Term Rewriting)* (8 + 6 = 14 points)

Part (a)

Let R be a term rewrite system and let $[t]$ denote the congruence class of the term t with respect to R. Prove: If R is confluent, and if x and y are two different variables, then $[x] \neq [y]$.

Part (b)

Give an example of a term rewrite system R such that R is locally confluent and $[x] = [y]$ for two different variables x and y.

Assignment 5 *(Critical Pairs, Termination)* (8 + 6 = 14 points)

Let E be the following set of equations over $\Sigma = (\{f/1,\ g/2,\ h/1,\ b/0,\ c/0\}, \emptyset)$.

$$\begin{aligned} g(x, h(x)) &\approx f(h(x)) && (1)\\ f(g(b, y)) &\approx f(h(y)) && (2)\\ h(h(z)) &\approx g(z, c) && (3) \end{aligned}$$

Part (a)

Suppose that the three equations in E are turned into rewrite rules by orienting them from left to right. Give all (non-trivial) critical pairs between the resulting three rules.

Part (b)

It is possible to orient the equations in E using an appropriate KBO, so that there are no critical pairs between the resulting rules. Give the weights and precedence of the KBO and explain how the equations are oriented.

Assignment 6 *(Dependency Pairs)* (6 + 8 = 14 points)

Part (a)

Let $\Sigma = (\Omega, \emptyset)$ with $\Omega = \{f/2,\ g/2,\ h/1,\ k/2,\ l/1,\ b/0\}$. Compute the dependency pairs of the following TRS R:

$$\begin{aligned} f(x, h(y)) &\to k(f(h(x), y), g(x, h(y))) && (1)\\ g(h(x), y) &\to h(f(x, y)) && (2)\\ g(x, b) &\to f(b, l(x)) && (3)\\ l(x) &\to h(x) && (4) \end{aligned}$$

Part (b)

Compute the approximated dependency graph (using cap and ren) for R and use the subterm criterion to show that R is terminating. If a graph is modified, depict both the original and the modified graph and indicate the strongly connected components in the graphs.