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Tutorials for "Automated Reasoning"
Exercise sheet 9

Exercise 9.1: *(3+3 P)*

(a) Show that the compactness theorem (Thm. 3.36) holds also for first-order logic with equality.

(b) Use the compactness theorem for first-order logic with equality to prove the following statement: Let F be a first-order formula with equality. If, for every natural number n, F has a model whose universe has at least n elements, then F has a model with an infinite universe.

Exercise 9.2: *(4 P)*
Let E be the following set of (implicitly universally quantified) equations over $\Sigma = (\{f/2,\ g/1,\ h/2,\ b/0,\ c/0\}, \emptyset)$:

$$\{\ f(x, g(y)) \approx h(g(x), y),\ \ f(b, x) \approx f(b, y),\ \ g(g(x)) \approx h(x, x)\ \}.$$

Determine for each Σ-term t in

$$\{\ f(g(z), g(z)),\ \ g(f(b, c)),\ \ f(c, b),\ \ g(g(g(g(b))))\ \}$$

the set of all t' such that $t \to_E t'$.

Exercise 9.3: *(4 P)*
Let $E = \{\, f(g(x)) \approx g(f(x))\,\}$. Give a derivation for $E \vdash f(f(g(g(y)))) \approx g(g(f(f(y))))$.

Exercise 9.4: *(4 P)*
Consider the signature $\Sigma = (\{f/1, b/0, c/0\}, \emptyset)$ and the set of (implicitly universally quantified) equations $E = \{\, f(f(x)) \approx x\,\}$. How many elements does the universe of $\mathrm{T}_\Sigma(\emptyset)/E$ have? How do they look like? How is $f_{\mathrm{T}_\Sigma(\emptyset)/E}$ defined on the universe?

Submit your solution during the tutorial on January 7 or 8 or in lecture hall E1.3, Room 001 during the lecture on January 8. Please write your name and the date of your tutorial group (Tue, Wed) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.