max planck institut
informatik

Universität
des
Saarlandes
FR Informatik

Uwe Waldmann **December 4, 2013**

Tutorials for "Automated Reasoning"
Exercise sheet 7

Exercise 7.1: (*5 P*)
Let $\succ$ be a total and well-founded ordering on ground atoms such that, if the atom A contains more symbols than B, then $A \succ B$. Let N be the following set of clauses:

$$\neg q(z, z)$$
$$\neg q(f(x), y) \vee q(f(f(x)), y) \vee p(x)$$
$$\neg p(a) \vee \neg p(f(a)) \vee q(f(a), f(f(a)))$$
$$p(f(x)) \vee p(g(y))$$
$$\neg p(g(a)) \vee p(f(f(a))$$

(a) Which literals are maximal in the clauses of N?

(b) Define a selection function *sel* such that N is saturated under $Res^{\succ}_{sel}$.

Exercise 7.2: (*5 P*)
Let N be the following set of ground clauses:

$$\neg P_3 \vee P_1 \vee P_1 \quad (1)$$
$$\neg P_2 \vee P_1 \quad (2)$$
$$P_4 \vee P_4 \quad (3)$$
$$P_3 \vee \neg P_2 \quad (4)$$
$$P_4 \vee P_3 \quad (5)$$

(a) Find a total atom ordering $\succ$ such that both clause (2) and (5) are redundant w.r.t. N.

(b) Prove that there is no atom ordering such that clause (4) is redundant w.r.t. N.

Exercise 7.3: (*4 P*)
Derive a maximal strict tableau for the set of formulas

$$P \to (Q \to R) \qquad (1)$$
$$P \to Q \qquad (2)$$
$$P \wedge \neg R \qquad (3)$$

Exercise 7.4: (*4 P*)
Refute the following set of formulas using one of the two variants of the tableau calculus for first-order formulas:

$$\forall x\, \exists y\, P(x, y) \qquad (1)$$
$$\exists z\, \forall w\, \neg P(f(z), w) \qquad (2)$$

(If you use tableaux with free variables, use $v_1, v_2, v_3, \ldots$ as names for free variables.)

Challenge Problem: (*4 Bonus Points*)
Prove part (ii) of Thm. 3.43: If N is a set of (not necessarily ground) clauses and $M \subseteq \mathit{Red}(N)$, then $\mathit{Red}(N) \subseteq \mathit{Red}(N \setminus M)$.

Submit your solution during the tutorial on December 10 or 11 or put it into the box in front of Björn Borowski's office (E1.5, Room 636) until December 11, 17:00. Please write your name and the date of your tutorial group (Tue, Wed) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.