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Tutorials for "Automated Reasoning"
Exercise sheet 4

Exercise 4.1: (*4 P*)
Prove that the process described in the *Getting Better Backjump Clauses* section terminates, provided that the clause $D \vee L$ that is applied in a resolution step is exactly the clause that allowed us to add the deduced literal L to M in a *Unit Propagate* or *Backjump* transition.

Exercise 4.2: (*6 P*)
Prove that the "Literal Elimination" rule explained in the *Preprocessing* section is satisfiability-preserving: Let N be a set of clauses in which the propositional variable P does not occur. For $1 \leq i \leq m$ let $C_i \vee P$ be a clause in which P occurs positively; for $1 \leq j \leq n$ let $D_j \vee \neg P$ be a clause in which P occurs negatively. Then

$$N \cup \{\, C_i \vee P \mid 1 \leq i \leq m \,\} \cup \{\, D_j \vee \neg P \mid 1 \leq j \leq n \,\}$$

is satisfiable if and only if

$$N \cup \{\, C_i \vee D_j \mid 1 \leq i \leq m,\, 1 \leq j \leq n \,\}$$

is satisfiable.

Exercise 4.3: (*4 P*)
Give OBDDs for the following three formulas:

(1) $\neg P$

(2) $P \leftrightarrow Q$

(3) $(P \wedge Q) \vee (Q \wedge R) \vee (R \wedge P)$

Exercise 4.4: (*4 P*)
Let the signature $\Sigma = (\Omega, \Pi)$ be given by $\Omega = \{+/2, s/1, 0/0\}$ and $\Pi = \emptyset$, and let

$$\begin{aligned} F_1 &= \forall x\,(x + 0 \approx x) \\ F_2 &= \forall x \forall y\,(x + s(y) \approx s(x + y)) \\ F_3 &= \forall x \forall y\,(x + y \approx y + x) \\ F_4 &= \neg \forall x \forall y\,(x + y \approx y + x). \end{aligned}$$

(1) Determine a Σ-algebra $\mathcal{A}$ with an universe of exactly two elements such that $\mathcal{A}$ is a model of F_1, F_2, F_3.

(2) Determine a Σ-algebra $\mathcal{A}$ with an universe of exactly two elements such that $\mathcal{A}$ is a model of F_1, F_2, F_4.

Challenge Problem: (*4 Bonus Points*)
What is the maximal number of interior (= non-leaf) nodes that an OBDD for a formula with four propositional variables can have?

Submit your solution during the tutorial on November 12 or 13 or in lecture hall E1.3, Room 001 during the lecture on November 13. Please write your name and the date of your tutorial group (Tue, Wed) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.