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## Tutorials for "Automated Reasoning" Exercise sheet 12

**Exercise 12.1:** (4 P)Suppose that we replace condition (1) in the definition of the Knuth-Bendix ordering by

(1')  $\operatorname{var}(t) \subseteq \operatorname{var}(s)$  and w(s) > w(t).

Show that the resulting relation is *not* a simplification ordering.

**Exercise 12.2:** (6 P) Apply the Knuth-Bendix procedure to the set of equations

 $\{f(f(x)) \approx g(x), f(a) \approx b\}$ 

and transform it into a finite convergent term rewrite system; use the Knuth-Bendix ordering with weight 1 for all function symbols and variables and the precedence g > f > a > b.

**Exercise 12.3:** (4 P) Apply the Knuth-Bendix procedure to the set of equations

 $\{f(0, f(x, f(y, z))) \approx f(f(0, x), f(y, z)), f(0, x) \approx 0, f(x, 1) \approx x\}$ 

and transform it into a finite convergent term rewrite system; use the Knuth-Bendix ordering with weight 1 for all function symbols and variables and the precedence f > 1 > 0. Start by orienting the first equation.

## **Exercise 12.4:** (4 P)

If the set R in the Knuth-Bendix completion procedure contains two rules whose left-hand sides are equal up to variable renaming, then none of them can be simplified using the other one. In practice, such a situation should never occur. Why?

## Challenge Problem: (4 Bonus Points)

Show that the relation  $\Box$  defined on page 109 of the script is well-founded.

Submit your solution during the tutorial on January 28 or 29 or in lecture hall E1.3, Room 001 during the lecture on January 29. Please write your name and the date of your tutorial group (Tue, Wed) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.