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Tutorials for "Automated Reasoning"
Exercise sheet 11

Exercise 11.1: *(6 P)*
Let $\Sigma = (\{f/1,\ g/2,\ h/1,\ b/0,\ c/0\}, \emptyset)$ and let

$$\begin{aligned} t_1 &= g(h(x), h(c)), \\ t_2 &= g(x, x), \\ t_3 &= g(b, f(x)), \\ t_4 &= f(g(x, y)), \\ t_5 &= h(g(x, c)). \end{aligned}$$

Determine for each $1 \le i < j \le 5$ whether t_i and t_j are uncomparable or comparable (and if so, which term is larger) with respect to

(a) a lexicographic path ordering with precedence $f > g > h > b > c$,

(b) a Knuth-Bendix-ordering with precedence $h > f > g > b > c$, where h has weight 0 and all other symbols have weight 1,

(c) a polynomial ordering over $\{\, n \in \mathbb{N} \mid n \ge 1 \,\}$ with $P_f(X_1) = X_1 + 1$, $P_g(X_1, X_2) = 2X_1 + X_2$, $P_h(X_1) = 3X_1$, $P_b = 1$ and $P_c = 3$.

Exercise 11.2: *(4 P)*

- Find a polynomial ordering $\succ$ over $\{\, n \in \mathbb{N} \mid n \ge 1 \,\}$ with linear polynomials such that $g(x) \succ x$, $h(x) \succ g(x)$, and $f(g(x)) \succ g(h(f(x)))$.
- Find a lexicographic path ordering $\succ$ such that $h(h(x)) \succ f(x)$ and $f(g(h(x), y)) \succ h(g(x, f(y)))$.

Exercise 11.3: *(4 P)*
Prove Thm. 4.30: If the precedence $\succ$ is total, then the lexicographic path ordering $\succ_{\text{lpo}}$ is total on ground terms, i.e., for all $s, t \in \mathrm{T}_\Sigma(\emptyset)$: $s \succ_{\text{lpo}} t \vee t \succ_{\text{lpo}} s \vee s = t$.

Exercise 11.4: *(4 P)*
Let $k_1, \ldots, k_n, k \in \mathbb{Z}$. Prove that $\sum k_i a_i + k > 0$ for all $a_1, \ldots, a_n \geq 1$ if and only if $k_i \geq 0$ for all $i \in \{1, \ldots, n\}$, and $\sum k_i + k > 0$.

Submit your solution during the tutorial on January 21 or 22 or in lecture hall E1.3, Room 001 during the lecture on January 22. Please write your name and the date of your tutorial group (Tue, Wed) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.