max planck institut
informatik

Universität
des
Saarlandes
FR Informatik

Uwe Waldmann **January 9, 2014**

Tutorials for "Automated Reasoning"
Exercise sheet 10

Exercise 10.1: (*4 P*)
Let $\Sigma = (\{f/2, b/0, c/0, d/0\}, \emptyset)$; let $E = \{\, \forall x\, (f(x,x) \approx b),\ c \approx d \,\}$; let $X = \{x, y, z\}$ be a set of variables. For any $t \in \mathrm{T}_\Sigma(X)$ let $[t]$ denote the congruence class of t w.r.t. E. Let $\mathcal{T} = \mathrm{T}_\Sigma(X)/E$ and let $\beta : X \to \mathcal{T}$ be the assignment that maps every variable to $[c]$.

State for each of the following "items" whether it is (a) a term, (b) a set of terms, (c) a set of ground terms, (d) a congruence class w.r.t. E, (e) a formula, (f) a truth value, or (g) non-sensical. (More than one answer may be correct.)

(1) $[c]$

(2) $[b]$

(3) $\{d\}$

(4) $f(c, d)$

(5) $[f(c, d)]$

(6) $f([c], [d])$

(7) $f_{\mathcal{T}}(\{c, d\}, \{c, d\})$

(8) $b \approx c$

(9) $\mathcal{T}(\beta)(\forall x, y\ (f(x, x) \approx f(y, y)))$

(10) $f_{\mathcal{T}}(\mathcal{T}(\beta)(x), \mathcal{T}(\beta)(y))$

Exercise 10.2: (*4 P*)
Prop. 4.10 states that, if $\to$ is normalizing and confluent, then $b \leftrightarrow^* c \ \Leftrightarrow\ b{\downarrow} = c{\downarrow}$. Prove it by induction on the length of the derivation without using the Church-Rosser Theorem 4.7.

Exercise 10.3: (*6 P*)
Compute all critical pairs for each of the following term rewrite systems:

(a) $\{\, f(g(f(x))) \to x,$
$\quad f(g(x)) \to g(f(x)) \,\}$

(b) $\{\, f(x,x) \to a,$
$\quad f(x, g(x)) \to b \,\}$

(c) $\{\, f(f(x,y),z) \to f(x, f(y,z)),$
$\quad f(x,1) \to x \,\}$

(d) $\{\, f(f(x,y),z) \to f(x, f(y,z)),$
$\quad f(1,x) \to x \,\}$

Which systems are locally confluent?

Exercise 10.4: (*4 P*)
Is the rewrite system

$$\{\, f(a) \to f(b),\ f(b) \to f(c),\ f(c) \to f(a),\ f(x) \to x \,\}$$

(i) terminating, (ii) normalizing, (iii) locally confluent, (iv) confluent? Give a brief explanation.

Challenge Problem: (*6 Bonus Points*)
Find a signature Σ containing at least one constant symbol, a set E of Σ-equations, and two terms $s, t \in \mathrm{T}_\Sigma(X)$ such that

$$\mathrm{T}_\Sigma(\{x_1\})/E \ \models\ \forall \vec{x}(s \approx t),$$

but

$$\mathrm{T}_\Sigma(\{x_1, x_2\})/E \ \not\models\ \forall \vec{x}(s \approx t)$$

where $\vec{x}$ consists of all the variables occurring in s and t. (The variables in $\vec{x}$ need not be contained in $\{x_1, x_2\}$.)

Submit your solution during the tutorial on January 14 or 15 or in lecture hall E1.3, Room 001 during the lecture on January 15. Please write your name and the date of your tutorial group (Tue, Wed) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.