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Tutorials for "Automated Reasoning"
Exercise sheet 1

Exercise 1.1: (*4 P*)
Find an abstract reduction system $(A, \to)$, such that $\to^+$ is irreflexive and $\to$ is normalizing, but not terminating.

Exercise 1.2: (*4 P*)
For an alphabet Σ with a well-founded ordering $>_\Sigma$ let the relation $\succ$ be defined as

$$w \succ w' :\Leftrightarrow |w| > |w'| \text{ or } (|w| = |w'| \text{ and } w >_{\Sigma,lex} w').$$

Prove that $\succ$ is a well-founded ordering on Σ^*.

Exercise 1.3: (*4 P*)
Let M be the set $\{a, b, c\}$. Determine an ordering $\succ$ of M such that the following statements hold for the multiset extension $(\succ_{\text{mul}})_{\text{mul}}$ of the multiset extension of $\succ$.

(1) $\big\{\{a, b\}, \{c\}\big\}\ (\succ_{\text{mul}})_{\text{mul}}\ \big\{\{a\}, \{b, c\}\big\}$ and

(2) $\big\{\{b\}, \{c, c\}\big\}\ (\succ_{\text{mul}})_{\text{mul}}\ \big\{\{b, b, b\}, \{c\}\big\}$.

Exercise 1.4: (*6 P*)
Let $(M, \succ)$ be an ordering and $a, b \in M$. We say that b is a *successor* of a, if $b \succ a$ and if there exists no $c \in M$ with $b \succ c$ and $c \succ a$.

Prove the following statements.

(1) If $(M, \succ)$ is well-founded, then every element of M has either a successor or it is maximal.

(2) If $(M, \succ)$ is well-founded and $\succ$ is total, then the successor is unique.

(3) If $(M, \succ)$ is well-founded, $\succ$ is total, and $|M| \geq 2$, then there are multisets over M which are neither minimal nor successors of any other multiset with respect to $\succ_{\text{mul}}$.

Submit your solution in lecture hall E1.3, Room 001 during the lecture on October 23. Please write your name and the date of your tutorial group (Tue, Wed) on your solution.

Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.