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**July 9, 2012**

**Tutorials for "Automated Reasoning"**
**Exercise sheet 13**

**Exercise 13.1:** (*3 P*)
Apply the Knuth-Bendix completion procedure to the set of equations $\{f(f(x)) \approx x, f(a) \approx b\}$ using the Knuth-Bendix ordering with weight 1 for all function symbols and variables. Use precedence $f > a > b$.

**Exercise 13.2:** (*3 P*)
Use the method described in Theorem 4.34 and unfailing completion to prove that the equation $f(g(x), x) \approx f(x, g(y))$ is a consequence of these equations:

$$\begin{aligned} f(x, y) &\approx f(y, z) \\ g(x) &\approx x. \end{aligned}$$

Use a KBO with constant weight 1 and $f > g$.

**Exercise 13.3:** (*2 P*)
Prove that the following term rewrite system is confluent:

$$\begin{aligned} f(g(x)) &\to x \\ g(f(x)) &\to x \\ f(b) &\to c \\ b &\to g(c). \end{aligned}$$

**Exercise 13.4:** (*2 P*)
Prove or disprove: every term rewrite system consisting of a single rule is confluent.

**Exercise 13.5:** (*3 P*)
For a term rewrite system $R$, we define $\mathrm{LSymb}(R)$ as the set of all function symbols ocurring

on the left hand side of the rewrite rules of $R$. Formally

$$\mathrm{LSymb}(R) = \bigcup_{l \to r \in R} \mathrm{symb}(l)$$

where $\mathrm{symb}(t)$ is a set of function (and constant) symbols in a term $t$. The union of two rewrite systems is just a system which rule set is the union of their rewrite rules. Prove or disprove: for two confluent rewrite systems $R_1$ and $R_2$ their union $R_1 \cup R_2$ is confluent if and only if $\mathrm{LSymb}(R_1) \cap \mathrm{LSymb}(R_2) = \emptyset$.

**Exercise 13.6:** (*2 Bonus Points*)
Show that if $E$ is a set of ground identities, then the instance rule of equational reasoning is redundant, i.e. if $E \Rightarrow^* s \approx t$ then there is a proof of $E \Rightarrow^* s \approx t$ that does not involve instantiation.

Submit your solution in lecture hall 001 during the lecture **on July 16**. Please write your name and the date of your tutorial group on your solution.

**Note:** Joint solutions are not permitted (work in groups is encouraged).