max planck institut
informatik

Universität
des
Saarlandes
FR Informatik

Marek Kosta
Christoph Weidenbach

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Tutorials for "Automated Reasoning"
Exercise sheet 11

Exercise 11.1: (*4 P*)
Refute the following set of clauses using the First-Order Superposition Theorem Prover. $S = \{P(a) \vee P(b),\ \neg P(x) \vee \neg P(f(x)) \vee Q(f(a)),\ \neg P(x) \vee P(f(x)),\ Q(a),\ \neg Q(f(x)) \vee \neg Q(x),\ Q(f(x)) \vee \neg P(x)\}$. Do so using strategy "simplify first", i.e. apply Clause Processing *only* when neither Tautology Deletion, nor any of Subsumption or Subsumption Resolution rules are applicable. After each step show how sets N, U, WO look like and which clauses were affected. Use KBO (weight function always one) with precedence $Q \succ P \succ f \succ a \succ b$.

Exercise 11.2: (*2 P*)
Prove the third part of the Proposition 3.34 from the lecture for terms.

Exercise 11.3: (*4 P*)
Encode the Post Correspondence Problem using the equality as the only predicate symbol, i.e. show how to construct for any instance I of the PCP the corresponding set of first-order formulas N such that N is unsatisfiable if and only if I has a solution and the only predicate symbol used in N is equality. Prove that your construction is correct.

Exercise 11.4: (*3 P*)
Let $\Sigma = \{f, a\}$ and $E := \{f(f(f(f(f(a))))) \approx a, f(f(f(a))) \approx a\}$. Prove that $f(a) \approx_E a$ using the inference system $\mathcal{I}$ from the lecture.

Exercise 11.5: (*3 P*)
Show that if the equation $(l \approx r) \in E$ and $var(l) \supseteq var(r)$ does *not* hold, then the rewrite relation $\rightarrow_E$ is not terminating.

Exercise 11.6: (*2 Bonus Points*)
A Σ-interpretation $\mathcal{A}$ is called term-generated, if for every $b \in U_{\mathcal{A}}$ there is a ground term $t \in T_\Sigma$ such that $b = \mathcal{A}(\beta)(t)$. Prove that a set of universally quantified equational clauses

has a model if and only if it has a term-generated model.

Submit your solution in lecture hall 001 during the lecture **on July 2**. Please write your name and the date of your tutorial group on your solution.

Note: Joint solutions are not permitted (work in groups is encouraged).