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Tutorials for "Automated Reasoning"
Exercise sheet 10

Exercise 10.1: *(4 P)*
Let a, b, c be constants, f binary function symbol and P and Q binary predicates. Apply the resolution calculus on the following two clauses:

$$\begin{aligned} C_1 &= \neg Q(f(x,z), f(y,z)) \vee Q(x,y) \vee Q(z,c) \vee \neg P(x,y) \\ C_2 &= \neg Q(f(a,c), b) \vee \neg P(a,x) \vee Q(a,b). \end{aligned}$$

How many different possibilities to apply the resolution calculus are there? Show all resulting clauses with respective substitutions.

Exercise 10.2: *(2 P)*
Let $\Sigma = \{a/0, b/0, f/1, g/1; P/1, Q/1\}$ be a signature (P, Q are predicates, the other symbols are function symbols). Find some Knuth-Bendix ordering (i.e. define weight function and precedence) in such a way that the following will hold: $Q(f(b)) \vee P(g(b)) \succ_{\text{kbo}} P(f(a)) \vee P(f(a)) \vee \neg P(g(b)) \succ_{\text{kbo}} P(g(b)) \vee P(a) \succ_{\text{kbo}} P(g(b)) \vee Q(b) \succ_{\text{kbo}} \neg P(f(a)) \vee Q(b) \succ_{\text{kbo}} \neg P(a)$. Note that for the ordering we consider the usual lifting of $\succ_{\text{kbo}}$ to literals (negative larger) and clauses (multiset extension). Justify your definitions.

Exercise 10.3: *(3 P)*
Let w be a weight function defined as follows on function symbols: $w(f) = 3, w(g) = 2, w(a) = 1$. On predicate symbols the function w is defined as $w(P) = 5, w(Q) = 4$. Weight of every variable is 1. Which literals in the following clauses are maximal when we consider Knuth-Bendix ordering induced by the weight function w? Mark all the maximal literals and justify your choice.

1. $\neg P(g(a)) \vee P(f(f(a)))$
2. $P(f(x)) \vee P(g(y))$
3. $\neg P(a) \vee \neg P(f(a)) \vee Q(f(a), f(f(a)))$

4. $\neg Q(f(x), y) \vee Q(f(f(x)), y) \vee P(x)$
5. $\neg P(x) \vee P(f(x))$
6. $Q(f(z)) \vee \neg P(f(g(x))) \vee \neg Q(g(f(f(y))))$

Exercise 10.4: (*2 P*)
Show some selection function sel such that the following set of clauses N is saturated with respect to the calculus $Sup_{\text{sel}}^{\succ}$ for some ordering $\succ$. $N = \{\neg P(x) \vee Q(f(a)) \vee P(f(x)),\ P(g(x)) \vee P(g(f(x))) \vee \neg P(a),\ P(g(x)) \vee Q(y) \vee P(f(f(f(y)))) \vee \neg Q(f(y)),\ P(g(y)) \vee \neg Q(f(a)),\ Q(f(z)) \vee P(f(g(f(z)))) \vee \neg Q(f(g(f(z))))\}$. Justify your definition of the selection function.

Exercise 10.5: (*3 P*)
Prove Lemma 3.28.

Exercise 10.6: (*3 Bonus Points*)
Consider a signature Σ. Let N be some set of ground clauses. Prove that for any ground clause C over the signature Σ, $N \models C$ if and only if $N \vdash_{Res} D$ for some clause D such that $D \subseteq C$.

Submit your solution in lecture hall 001 during the lecture **on June 25**. Please write your name and the date of your tutorial group on your solution.

Note: Joint solutions are not permitted (work in groups is encouraged).