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Tutorials for "Automated Reasoning"
Exercise sheet 9

Exercise 9.1: (*4 P*)
Transform the formula

$$\neg\forall w \exists x \neg\forall y\, (P(w,x,y) \leftrightarrow \forall z P(z,y,x))$$

into clausal normal form. Pick and use suitable algorithm from the lecture.

Exercise 9.2: (*4 P*)
Let $\varphi = \exists z \forall x\, [(P(x,z) \wedge \forall y(P(x,y) \rightarrow P(y,z))) \rightarrow P(z,x)]$ be first-order formula. Skolemize φ using the procedure from section 3.6 ("Getting Small Skolem Functions").

Exercise 9.3: (*2 P*)
Compute the $mgu(E)$, where $E = \{f(g(x,x)) \doteq y; h(y) \doteq h(v); v \doteq f(g(z,w))\}$.

Exercise 9.4: (*4 P*)
Refute the following set of clauses using the superposition for general clauses. $S = \{P(a) \vee P(b),\ \neg P(x) \vee \neg P(f(x)) \vee Q(f(a)),\ \neg P(x) \vee P(f(x)),\ Q(a),\ \neg Q(f(x)) \vee \neg Q(x),\ Q(f(x)) \vee \neg P(x)\}$.

Exercise 9.5: (*6 Bonus Points*)
Let T be a first-order theory (with equality). Prove that if T has models of arbitrarily big finite cardinality then T has also a model of infinite cardinality. Property "has models of arbitrarily big finite cardinality" is: For every natural number n there is another natural number $n' > n$ such that T has a model of cardinality n'.

Submit your solution in lecture hall 001 during the lecture **on June 18**. Please write your name and the date of your tutorial group on your solution.

Note: Joint solutions are not permitted (work in groups is encouraged).