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Tutorials for "Automated Reasoning"
Exercise sheet 3

Exercise 3.1: (*4 P*)
Determine which of the following formulas are valid/satisfiable/unsatisfiable:

1. $(P \wedge Q) \rightarrow (P \vee Q)$
2. $(P \vee Q) \rightarrow (P \wedge Q)$
3. $\neg(P \wedge \neg\neg P)$
4. $\neg(\neg P \vee \neg\neg P)$
5. $((P \rightarrow Q) \wedge (\neg P \rightarrow R)) \rightarrow (Q \vee R)$
6. $P \rightarrow (Q \rightarrow P)$
7. $(\neg Q \rightarrow \neg P) \rightarrow (P \rightarrow Q)$
8. $(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$

Exercise 3.2: (*3 P*)
Logical connective NAND (notation: $\uparrow$) is defined as:

$$\phi \uparrow \psi \leftrightarrow \neg(\phi \wedge \psi).$$

Show how connectives $\neg$, $\wedge$ and $\vee$ can be equivalently rewritten using only connective $\uparrow$.

Exercise 3.3: (*2 P*)
Let N be a set of propositional formulas and ϕ be one propositional formula. Prove that $N \models \phi$ if and only if $N \cup \{\neg\phi\}$ is not satisfiable (i.e. $N \cup \{\neg\phi\}$ has no model, notation: $N \cup \{\neg\phi\} \models \bot$).

Exercise 3.4: (*2 P*)
Let ϕ and ψ be two propositional formulas. Prove or disprove:

1. If ϕ is satisfiable or ψ is satisfiable then $\phi \lor \psi$ is satisfiable.
2. If ϕ is satisfiable and ψ is satisfiable then $\phi \land \psi$ is satisfiable.

Disprove means: provide a counter example and show that it is really a counter example.

Exercise 3.5: (*4 Bonus Points*)
Let Σ be a non-empty finite signature (set of propositional variables). As we already know we can percieve propositional formulas over Σ as functions from $\{0,1\}^{|\Sigma|}$ to $\{0,1\}$. Compare following sets of functions:

1. all functions from $\{0,1\}^{|\Sigma|}$ to $\{0,1\}$
2. functions representable by a propositional formula with connectives $\land$, $\lor$
3. functions representable by a propositional formula with connectives $\land$, $\lor$, $\rightarrow$
4. functions representable by a propositional formula with connectives $\land$, $\neg$

Compare means: for every pair of the above mentioned sets decide and prove which relation $\subseteq$, $\supseteq$ (or incomparable) holds.

Submit your solution in lecture hall 001 during the lecture on May 7. Please write your name and the date of your tutorial group on your solution.

Note: Joint solutions are not permitted (work in groups is encouraged).