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Tutorials for “Automated Reasoning”
Exercise sheet 2

Exercise 2.1: (3 P)

Let $(\mathbb{N}^+ - \{1\}, <_d)$ be the set of positive natural numbers without number one ordered by the relation of divisibility $<_d$ defined by $a <_d b$ if a divides b . Are there minimal elements? Is there a smallest element? How do they look like?

Exercise 2.2: (3 P)

Let $(\mathbb{Q}, <)$ be the set of rational numbers ordered with the usual ordering relation $<$. Construct subsets of \mathbb{Q} with following properties (for each item one subset):

- (a) the set **is** well founded and **has** a minimal element
- (b) the set **is not** well founded and **has** a minimal element
- (c) the set **is** well founded and **has** a maximal element
- (d) the set **is** well founded and does **not have** a maximal element

Exercise 2.3: (4 P)

Let $(M_1, >_1)$; $(M_2, >_2)$ and $(M_3, >_3)$ be three strictly partially ordered sets. Let $\varphi : M_1 \rightarrow M_2$ and $\psi : M_2 \rightarrow M_3$ be two monotone mappings from M_1 to M_2 and from M_2 to M_3 , respectively. Prove or disprove: Composition of these two mappings is also a monotone mapping.

Bonus Problem (2 Bonus Points)

Give a counterexample to the reverse implication of Lemma 1.10.

Submit your solution in lecture hall 001 during the lecture on April 30. Please write your name and the date of your tutorial group on your solution.

Note: Joint solutions are not permitted (work in groups is encouraged).