UNIVERSITÄT DES SAARLANDES
FR 6.2 – Informatik
Christoph Weidenbach

Lecture "Automated Reasoning"
(Summer Term 2012)

Final Final Examination

Name: ....

Student Number: ....

Some notes:

- Things to do at the beginning:

  Put your student card and identity card (or passport) on the table.
  Switch off mobile phones.
  Whenever you use a new sheet of paper (including scratch paper), first write your name and student number on it.

- Things to do at the end:

  Mark every problem that you have solved in the table below.
  Stay at your seat and wait until a supervisor staples and takes your examination text.
  Note: Sheets that are accidentally taken out of the lecture room are invalid.

Sign here: **Good luck!**

....

Problem	1	2a	2b	2c	3	4	5	6	7	Σ
Answered?										
Points										

Problem 1 *(DPLL)* (6 points)

Check via the rule-based CDCL calculus ($\Rightarrow_{\text{DPLL}}$ + learning rule) whether the following clause set is satisfiable or not. Learn backjump clauses.

$$\{P1 \lor Q \lor R \lor P2, \neg P2 \lor P1 \lor R, \neg P1 \lor R, \neg P1 \lor \neg R, P2 \lor \neg Q, \neg R \lor P1\}$$

Problem 2 *(Superposition Model Building)* (8 points)

Consider the following clause set N with respect to an LPO where $g \succ f \succ b \succ a$.

$N = \{f(a, g(a)) \approx g(a), f(a, a) \approx a \lor f(a, a) \approx g(a), b \not\approx g(b), f(a, g(a)) \approx g(b), b \not\approx a\}$

(a) Compute R_∞.

(b) Determine the minimal false clause.

(c) Compute the superposition inference out of (b), add it to the clause set N compute the new respective R_∞.

Problem 3 *(Unification)* (6 points)

Check whether the unification problem below has a solution using $\Rightarrow_{PU}$. If it has, present the unifier. The symbols x, y, z, u are all variables.

$$E = \{f(x, g(x)) \doteq f(f(z, y), u), g(z) \doteq y, g(z) \doteq g(g(u))\}$$

Problem 4 *(CNF)* (6 points)

Apply the CNF algorithm of Section 3.6 from the lecture plus the eventual transformation to clauses to the first-order formula below. There is no beneficial subformula to rename.

$$\forall x, z \, \exists y \; (P(x, z) \rightarrow (Q(x, y) \lor (P(z, y) \land R(x, z))))$$

Problem 5 *(Completion)* (8 points)

Apply completion ($\Rightarrow_{\mathrm{KBC}}$) to the following set of equations with respect to a KBO where all signature symbols (and variables) have weight 1 and $f \succ g$ and x, y are variables.

$$N = \{f(x,x) \mathrel{\dot{\approx}} x, f(g(y),y) \mathrel{\dot{\approx}} g(y), g(g(x)) \mathrel{\dot{\approx}} g(x)\}$$

Problem 6 *(Saturation)* (6 points)

Determine an ordering and a selection function such that for the clause set below no superposition inference is possible that is not a tautology. As usual, a is a constant and x, y are variables. Show the maximal/selected literals and argue why there is no inference.

$$N = \{f(y,x) \approx x \vee f(h(y,y),x) \approx x, f(x,x) \approx g(x), f(x,y) \approx g(y) \vee f(a,f(a,y)) \not\approx y\}$$

Problem 7 *(Superposition Termination)* (7 points)

Let N be a finite set of predicative first-order clauses, i.e., there ore only predicative atoms and no equations. For example, a clause like $P(x, y) \vee \neg R(x, g(x)) \vee \neg R(y, y)$. For a clause C, let C^+ be the positive literals of C and C^- be the negative ones, respectively. Let us assume that the maximal depth of any variable x in C^- is larger than the maximal depth of x in C^+. Furthermore, we assume for all $C \in N$ that $\mathrm{vars}(C^+) \subseteq \mathrm{vars}(C^-)$.

Prove that a proper instantiation of the superposition calculus can finitely saturate N.