Assignment 1 *(CNF)* (10 points)

Transform the formula

$$\neg\,\forall w\,\exists x\,\neg\,\forall y\,\Big(P(w,x,y) \leftrightarrow \forall z\,P(z,y,x)\Big)$$

into clause normal form.

Assignment 2 *(First-Order Logic)* (14 points)

Let $\Sigma = (\Omega, \Pi)$ be a signature; let $\Sigma' = (\{c/0\}, \Pi)$. For any Σ-formula F let $\mathrm{rep}_c(F)$ be the Σ'-formula obtained from F by replacing every non-variable argument of a predicate (including $\approx$) by the constant symbol c. (E.g., if F is $\forall x\,\exists y\,P(f(y), x, b) \vee f(x) \approx y$, then $\mathrm{rep}_c(F)$ is $\forall x\,\exists y\,P(c, x, c) \vee c \approx y$.) Prove: If F is valid, then $\mathrm{rep}_c(F)$ is valid.

Assignment 3 *(Redundancy)* (10 points)

Give an example of two different ground first-order clauses without equality F and G, such that F entails G and G is *not* redundant w. r. t. $\{F\}$. If necessary, specify the atom ordering.

Assignment 4 *(Critical Pairs, Termination)* (8 + 8 = 16 points)

Let E be the following set of equations over $\Sigma = (\{f/1,\ g/1,\ h/2,\ b/0\}, \emptyset)$.

$$h(f(b), f(x)) \approx f(h(b, x)) \qquad (1)$$
$$g(g(x)) \approx f(g(x)) \qquad (2)$$
$$g(h(x, x)) \approx h(g(x), x) \qquad (3)$$

Part (a)

Suppose that all equations in E are turned into rewrite rules by orienting them from left to right. Give all (non-trivial) critical pairs between the resulting rules.

Part (b)

It is possible to orient the equations in E using an appropriate LPO, so that there are no critical pairs between the resulting rules. Give the precedence of the LPO and explain how the equations are oriented.

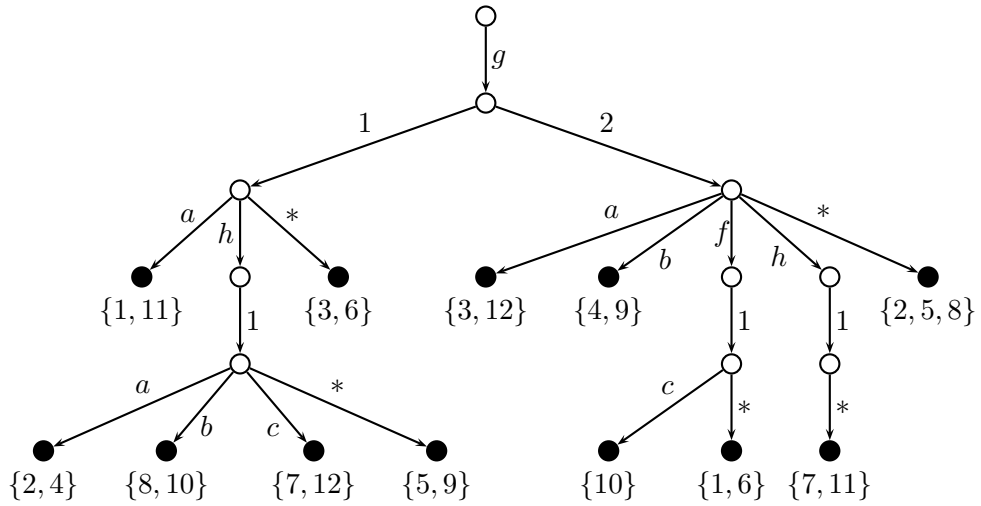
Assignment 5 *(Path Indexing)* (6 + 6 = 12 points)

Part (a)

Does the path index below contain the terms $t_1 = g(h(c), h(*))$, $t_2 = g(h(a), b)$, $t_3 = g(*, *)$? If yes, what are their numbers in the index?

Part (b)

Find the numbers of all terms in the path index that are generalizations of $s = g(h(a), f(*))$ (that is, terms t such that $s = t\sigma$).



Assignment 6 *(Dependency Pairs)* (6 + 6 + 6 = 18 points)

Part (a)

Let $\Sigma = (\Omega, \emptyset)$ with $\Omega = \{f/2,\ g/2,\ h/1,\ k/1,\ l/1,\ b/0\}$. Compute the dependency pairs of the following TRS R:

$$f(x, x) \to h(g(x, k(l(x)))) \quad (1)$$
$$g(h(x), y) \to f(h(y), x) \quad (2)$$
$$k(g(x, x)) \to k(b) \quad (3)$$
$$l(h(x)) \to h(x) \quad (4)$$

Part (b)

Compute the approximated dependency graph for R and use it to show that R is terminating.

Part (c)

For R, the exact dependency graph and the approximated dependency graph as defined in Sect. 6.1 of the lecture do *not* agree. Where do they differ?