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Tutorials for "Automated Reasoning"
Exercise sheet 5

Exercise 5.1: (*5 P*)
Solve exercise 4.2 from sheet 4.

Exercise 5.2: (*3 P*)
Which of the following closed formulas are valid, satisfiable, unsatisfiable? Explain your decision.

a) $\forall x\, p(x) \rightarrow \exists x\, p(x)$

b) $\forall x\, (p(x) \rightarrow p(f(x))) \wedge p(b) \wedge \neg p(f(f(b)))$

c) $[\forall x\, (p(x) \rightarrow p(f(x))) \wedge p(b)] \rightarrow \forall x\, p(x)$

d) $\exists x\, p(x) \rightarrow p(b)$

e) $[\forall x\, (p(x) \rightarrow p(f(x)))] \rightarrow \exists x\, p(x)$

f) $[\forall x\, (p(x) \rightarrow p(f(x))) \wedge p(b)] \rightarrow \forall x\, \neg p(x)$

Exercise 5.3: (*3 P*)
Prove Proposition 3.4 from the lecture by structural induction.

Exercise 5.4: (*2 P*)
Prove or refute the following statements:

a) If F is a first-order formula, then F is valid if and only if $F \rightarrow \bot$ is unsatisfiable.

b) If F is a first-order formula and x a variable, then F is unsatisfiable if and only if $\exists x\, F$ is unsatisfiable.

Challenge Problem: (*2 Bonus Points*)
Prove that a formula $\exists x\, F$ is satisfiable iff $F[a/x]$ is satisfiable where a is a constant that does not occur in F.

Submit your solution in lecture hall 002 during the lecture on May 25. Please write your name and the date of your tutorial group (Tue, Wed, Fri) on your solution.

Note: Joint solutions are not permitted (work in groups is encouraged).