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Tutorials for “Automated Reasoning”
Exercise sheet 4

Exercise 4.1: (3 P)

Use the transition system $\Rightarrow_{\text{DPLL}(T)}$ for the DPLL(T) procedure to check whether the following ground LA formula F is satisfiable or not. For each step explain which rule you use and why. Use the Fourier-Motzkin procedure to check satisfiability of a conjunction of LA literals.

$$F = x \leq 2 \wedge \neg(y > 4 \vee \neg(x + y < 10 \rightarrow 2x + 3y \geq 30))$$

Exercise 4.2: (5 P)

Delivery postponed by one week, submit May 25

Interpret the following predicates and functions as indicated, and translate the given English expressions (FOL formulas) into FOL formulas (English expressions).

$H(x)$ means “ x is a horse”, $tail(x)$ means “the tail of x ”, $mane(x)$ means “the mane of x ”, $B(x)$ means “ x is in the barn”, $W(x)$ means “ x is white”, $K(x)$ means “ x is black”, $L(x, y)$ means “ x likes y ”.

- “Every horse in the barn which has a white tail has a black mane”.
- “White horses do not like horses with black manes”.
- “No horse in the barn has a white tail”.
- $\exists x (B(x) \wedge (\forall y ((B(y) \wedge (K(tail(y)))) \rightarrow L(x, y))))$.
- $\neg \exists x (B(x) \wedge \neg W(tail(x)))$.

Exercise 4.3: (4 P)

Let F and σ be a first-order formula and a substitution, respectively, as given below. Compute $F\sigma$ and identify the free and bound variables of F and $F\sigma$.

a) $F = \neg(\forall x P(x, z) \wedge \neg\exists y (Q(x, y) \vee \forall z \neg R(y, z))) \rightarrow \exists z Q(x, z)$,
 $\sigma = [f(x, y)/x, a/y, g(f(u))/z, b/u]$.

b) $F = \forall x \exists y R(f(x, y), c) \rightarrow \exists z S(y, z)$,
 $\sigma = [f(c, c)/x, b/y, g(f(u, c))/z, x/u]$.

Challenge Problem: (2 Bonus Points)

A Horn clause is a clause with at most one positive literal. Prove that $\Rightarrow_{\text{DPLL}}$ without the rule Decide is a decision procedure for Horn clause sets.

Submit your solution in lecture hall 002 during the lecture on May 18. Please write your name and the date of your tutorial group (Tue, Wed, Fri) on your solution.

Note: Joint solutions are not permitted (work in groups is encouraged).