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Tutorials for "Automated Reasoning"
Exercise sheet 1

Exercise 1.1: (*2 P*)
Let F, F', G, G' be propositional formulas. Prove or refute the following propositions using truth tables:

1. If $F \rightarrow (G \leftrightarrow G')$ and $G \rightarrow (F \leftrightarrow F')$ then $(F \wedge F') \leftrightarrow (G \wedge G')$ also holds.
2. If $F \rightarrow (G \leftrightarrow G')$ and $G \rightarrow (F \leftrightarrow F')$ then $(F \wedge G) \leftrightarrow (F' \wedge G')$ also holds.

Exercise 1.2: (*3 P*)
Determine which of the following formulas are valid/satisfiable/unsatisfiable (don't use truth tables):

(1) $(P \wedge Q) \rightarrow (P \vee Q)$

(2) $(P \vee Q) \rightarrow (P \wedge Q)$

(3) $(\neg P \rightarrow Q) \rightarrow ((\neg P \rightarrow \neg Q) \rightarrow P)$

(4) $\neg(P \rightarrow \neg P)$

(5) $\neg\big(P \vee \neg(P \wedge Q)\big)$

(6) $(P \vee \neg Q) \wedge \neg(\neg P \rightarrow \neg Q)$

Exercise 1.3: (*4 P*)
Let F, G be propositional formulas and P be a propositional variable which does not occur in F nor in G. Prove or refute the following propositions:

1. If $F \wedge G$ is valid/satisfiable, then $P \wedge G \wedge (P \rightarrow F)$ is valid/satisfiable.

2. Let G be unsatisfiable and $F \models G$. Then $F \vee G$ satisfiable.
3. If $F \rightarrow G$ is valid, and $G \rightarrow H$ is satisfiable, then $F \rightarrow H$ is satisfiable.
4. If F is satisfiable and G is satisfiable, then $F \wedge G$ is satisfiable.

Exercise 1.4: (*2 P*)
Transform the following formula to both CNF and DNF following the conversion steps from the lecture: $((P \rightarrow Q) \vee R) \wedge (\neg Q \rightarrow P)$.

Exercise 1.5: (*1 P*)
Let F be a propositional formula. Show how to check its validity using an implementation of the DPLL procedure.

Challenge Problem: (*2 Bonus Points*)
Let F be a propositional formula which contains no occurrence of $\rightarrow$ or $\leftrightarrow$, then F° is the propositional formula obtained by replacing all occurrences of propositional variables by their negations.

The *dual* of F, which we denote here by F^*, is the propositional formula obtained by replacing every occurrence of $\top$ by $\bot$, every occurrence of $\bot$ by $\top$, every occurrence of $\vee$ by $\wedge$ and every occurrence of $\wedge$ by $\vee$.

Prove or refute that $F^* \models\!\mid \neg F^\circ$.

Submit your solution in lecture hall 002 during the lecture on April 27. Please write your name and the date of your tutorial group (Tue, Wed, Fri) on your solution.

Note: Joint solutions are not permitted (work in groups is encouraged).