UNIVERSITÄT DES SAARLANDES
FR 6.2 – Informatik
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Lecture "Automated Reasoning"
(Summer Term 2010)

First Final Examination

Name: ....

Student Number: ....

Some notes:

- Things to do at the beginning:

  Put your student card and identity card (or passport) on the table.
  Switch off mobile phones.
  Whenever you use a new sheet of paper (including scratch paper), first write your name and student number on it.

- Things to do at the end:

  Mark every problem that you have solved in the table below.
  Stay at your seat and wait until a supervisor staples and takes your examination text.
  Note: Sheets that are accidentally taken out of the lecture room are invalid.

Sign here: **Good luck!**

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Problem	1	2	3a	3b	3c	4	5	6	7	Σ
Answered?										
Points										

Problem 1 *(DPLL)* (6 points)

Figure out whether the following set N of clauses is satisfiable by an application of the $\Rightarrow_{\mathrm{DPLL}}$ calculus. Explicitely present backjump clauses if needed.

$$N = \{A \vee B,\ P \vee Q,\ \neg A \vee \neg P,\ \neg B \vee \neg Q,\ \neg A \vee \neg Q,\ \neg P \vee A\}$$

Problem 2 *(Unification)* (6 points)

Figure out using $\Rightarrow_{PU}$ whether the following equation has a solution.

$$f(g(x), h(y, g(x))) \doteq f(g(y), h(g(z), z))$$

Problem 3 *(Orderings)* (2+2+2=6 points)

For both orderings, KBO and LPO, either give an instantiation of the ordering for every pair that orders the terms left to right (left one larger), or justify why the terms cannot be ordered that way by the respective ordering.

a) $f(g(x), f(x, y)),\ g(h(h(x, a), h(x, y)))$

b) $a,\ f(b, b)$

c) $f(y, g(h(x, y))),\ f(h(x, y), h(x, y))$

Problem 4 *(Ordered Resolution)* (6 points)

Consider the clause set below. Define an ordering (KBO or LPO) and a selection function on the literals and then saturate the clause set using this setting by the ordered resolution calculus. Mark maximal/selected literals.
Hint: try to minimize the number of potential ordered resolution steps.

$$N = \{\neg R(x,y) \lor R(g(x),y),\ T(x) \lor S(x),\ \neg S(x) \lor R(x,z) \lor \neg R(x,y) \lor \neg R(y,z)\}$$

Problem 5 *(Completion)* (10 points)

Apply Knuth-Bendix completion to the following set of equations. Start with a KBO where all symbols have weight 1 and $f \succ h \succ g \succ a$. Stick to the rules of the algorithm.

$$E = \{f(x,y) \approx h(x,y),\ f(g(x),y) \approx f(x,y),\ g(a) \approx a\}$$

Problem 6 *(Superposition)* (6 points)

Consider the following two clauses with respect to a KBO with $f \succ g \succ b \succ a$ and constant weight 1 for all symbols. Mark the maximal literals (no selection) and compute all possible superposition inferences between the two clauses.

$$f(g(x), z) \approx f(x, a) \lor f(x, y) \approx g(x)$$
$$f(g(a), x) \not\approx a \lor f(g(b), x) \approx b$$

Problem 7 *(Decidability)* (2 + 4 = 6 points)

Let N be a finite set of Horn clauses, saturated by superposition. All clauses in N are of the form $C_i = C'_i \vee l_i \approx r_i$ where $l_i \approx r_i$ is strictly maximal in C_i, $l_i \succ r_i$, $\text{vars}(r_i) \subseteq \text{vars}(l_i)$, $\text{vars}(C'_i) \subseteq \text{vars}(l_i)$, and the C'_i only contain disequations or they may be empty.

a) Prove that N is satisfiable.

b) Given a ground disequation $s \not\approx t$, show that the unsatisfiability of the clause set $N \cup \{s \not\approx t\}$ is decidable. *Hint:* show that saturation always terminates, consider the clause ordering.