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Tutorials for "Automated Reasoning"
Exercise sheet 9

Exercise 9.1: (*2 P*)
Let $E = \{ f(g(x)) \approx g(f(x)) \}$. Give a derivation for $E \vdash f(f(g(g(y)))) \approx g(g(f(f(y))))$.

Exercise 9.2: (*4 P*)
Is the rewrite system

$$\{ f(a) \to f(b),\ f(b) \to f(c),\ f(c) \to f(a),\ f(x) \to x \}$$

(i) terminating, (ii) normalizing, (iii) locally confluent, (iv) confluent? Give a brief explanation.

Exercise 9.3: (*3 P*)
Prove that if $E \vdash s \approx t$ then $s \leftrightarrow_E^* t$.
(This is the part (ii) $\Rightarrow$ (i) in the proof of Lemma 4.11.)

Exercise 9.4: (*3 P*)
Let E be a set of equations, let $\theta : X \to T_\Sigma(X)$ be a substitution. Prove that $E \vdash t \approx t'$ implies $E \vdash t\theta \approx t'\theta$ for all terms t, t' over Σ.

Challenge Problem: (*2 Bonus Points*)
A relation $\rightarrow$ is *strongly confluent* (for all x, y_1, y_2) iff

$$y_1 \leftarrow x \rightarrow y_2 \Rightarrow \exists z.\ y_1 \rightarrow^* z \leftarrow^= y_2$$

Does the strong confluence imply the following property?

$$y_1 \leftarrow x \rightarrow y_2 \Rightarrow \exists z.\ y_1 \rightarrow^= z \leftarrow^= y_2$$

Give a proof or counterexample.

Submit your solution in lecture hall 002 during the lecture on June 19. Please write your name and the date of your tutorial group (Mon, Tue, Thu) on your solution.

Note: Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.