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May 20, 2008

Tutorials for “Automated Reasoning”
Exercise sheet 6

Exercise 6.1: (*2 P*)
Prove that I_N is a minimal model in the following sense. For every ground clause set N with $I_N \models N$, there exists no Herbrand interpretation J with $J \subset I_N$ and $J \models N$.

Exercise 6.2: (*2 P*)
Use the transition system $\Rightarrow_{\mathrm{DPLL}}$ for the DPLL procedure to check whether the following set of propositional clauses is satisfiable or not. For each step explain briefly which rule you use and why.

$$\begin{array}{rcrcrcrcrl}
A & \vee & B & \vee & C & & & & & (1)\\
\neg A & \vee & B & & & & & & & (2)\\
A & & & \vee & \neg C & \vee & \neg D & \vee & \neg E & (3)\\
A & \vee & \neg B & & & & & & & (4)\\
\neg A & \vee & \neg B & & & & & & & (5)\\
 & & & & \neg C & \vee & D & \vee & E & (6)\\
 & & & & & & \neg D & \vee & E & (7)\\
A & & & & & \vee & D & \vee & \neg E & (8)
\end{array}$$

Exercise 6.3: (*2 P*)
Which of the following closed formulas are valid, satisfiable, unsatisfiable? Give concrete Σ-algebras if the formula is satisfiable or a proof using ground resolution otherwise.

a) $[\exists x\,(p(x) \rightarrow p(f(x)))] \rightarrow p(f(b))$

b) $[\exists x\,(p(x) \rightarrow \neg p(f(x)))] \rightarrow \forall x\, p(f(x))$

c) $\forall x\,(p(x) \rightarrow p(f(x))) \wedge p(b) \wedge \neg p(f(f(b)))$

d) $[\forall x\,(p(x) \rightarrow p(f(x)))] \rightarrow \exists x\, p(x)$

Exercise 6.4: (*3 Bonus Points*)
Use the transition system $\Rightarrow_{\mathrm{DPLL}(T)}$ for the DPLL(T) procedure to check whether the following formula F is satisfiable or not. For each step explain briefly which rule you use and why. Check the T-(in)consistency of a valuation by using the Fourier-Motzkin algorithm.

$$F = \neg(((x \le 5 \lor x < y) \land x < 7 \land x' = x + 1 \land y \le 6) \to x' < 7)$$

Exercise 6.5: (*4 P*)
Solve the following unification problems using both $\Rightarrow_{SU}$ and $\Rightarrow_{PU}$.

a) $E_1 = \{f(g(x), x, g(u), u) \doteq f(y, g(z), z, g(w))\}$

b) $E_2 = \{f(g(h(x), w), h(x), g(w, h(k(a)))) \doteq f(y, h(z), y)\}$

Challenge Problem: (*2 Bonus Points*)
We call a term t *linear* if any variable occurs at most once in t. We define the *depth* of a term t to be $max\{|p| \mid p \in pos(t)\}$. Now consider two linear terms s, t that do not share variables. Prove that for any mgu σ of s and t we have

$$max(depth(t), depth(s)) = max(depth(t\sigma), depth(s\sigma))$$

Submit your solution in lecture hall 002 during the lecture on May 29. Please write your name and the date of your tutorial group (Mon, Tue, Thu) on your solution.

Note: Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.