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May 15, 2008

Tutorials for "Automated Reasoning"
Exercise sheet 5

Exercise 5.1: (*4 P*)
Prove the validity of the formula

$$[\forall x\,((p(x) \to q(x)) \wedge (q(x) \to r(x)))] \to [\forall x\,(p(x) \to r(x))]$$

via a ground resolution refutation. First, build the relevant CNF, second generate the necesssary ground clauses for the third, final ground resolution refutation.

Exercise 5.2: (*3 P*)
Let $\succ$ be the total ordering on the ground atoms $D \succ C \succ B \succ A$.

a) For each of the following pairs of triples, determine the lexicographic ordering relationship between the first and the second triple: $((A, A, C)\ ?\ (A, B, C))$, $((D, A, C)\ ?\ (B, C, D))$, $((B, A, A)\ ?\ (A, C, C))$ generated by the lexicographic extension of $\succ$.

b) Consider the triples of a) as multisets and determine the relationship generated by the multiset extension of $\succ$.

Exercise 5.3: (*3 P*)
Let N be the clause set $\{A \vee B \vee C,\ \neg B \vee C,\ B \vee \neg C \vee A,\ \neg A \vee B\}$ and $\succ$ as in Exercise 5.2.

a) Determine I_N.

b) Which clause is false in I_N?

c) Show the resolution step yielding a smaller counterexample.

Challenge Problem: (*2 Bonus Points*)
Consider renaming in first-order logic. Let $F/p = G$ be a formula where the subformula G has positive polarity and occurs at position p in F. The variables $x_1, \ldots, x_n$ occur freely in G and P is a fresh predicate.

a) Prove the following statement:

F is satisfiable iff $\forall x_1, \ldots, x_n(P(x_1, \ldots, x_n) \rightarrow G) \wedge F[P(x_1, \ldots, x_n)]_p$ is satisfiable.

b) Why is the renaming not validity preserving?

Submit your solution to Manuel Lamotte, building E14, office 115F until May 26, 9 am (office hours on workdays 7.30 - 15.00). Please write your name and the date of your tutorial group (Mon, Tue, Thu) on your solution.

Note: Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.