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Tutorials for “Automated Reasoning”
Exercise sheet 4

Exercise 4.1: (3 P)

Which of the following closed formulas are valid, satisfiable, unsatisfiable? Explain your decision.

- a) $\exists x p(x) \rightarrow \forall x p(x)$
- b) $\forall x p(x) \rightarrow p(b)$
- c) $\forall x (p(x) \rightarrow p(f(x))) \wedge \neg p(b) \wedge p(f(f(b)))$
- d) $[\forall x (p(x) \rightarrow \neg p(f(x))) \wedge p(b)] \rightarrow \exists x \neg p(x)$
- e) $[\forall x (p(x) \rightarrow p(f(x))) \wedge p(b)] \rightarrow \forall x \neg p(x)$
- f) $[\forall x (p(x) \rightarrow p(f(x)))] \rightarrow \exists x p(f(x))$

Exercise 4.2: (3 P)

Let F and G be first-order formulas over Σ , let x be a variable, and let y be a variable not occurring in F , G , and x . Show that

$$\mathcal{A}(\beta)((\forall x F) \wedge G) = \mathcal{A}(\beta)(\forall y (F[y/x] \wedge G))$$

holds for all Σ -algebras \mathcal{A} and for all assignments $\beta : X \rightarrow U_{\mathcal{A}}$.

Exercise 4.3: (4 P)

Show Proposition 3.8 from the lecture.

Challenge Problem: (*2 Bonus Points*)

What do you need in order to build formulas which have only infinite models?

Submit your solution in lecture hall 002 during the lecture on May 15. Please write your name and the date of your tutorial group (Mon, Tue, Thu) on your solution.

Note: Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.