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Tutorials for “Automated Reasoning”
Exercise sheet 2

Exercise 2.1: (2 P)

Prove: A formula in DNF is unsatisfiable, if and only if each of its conjunctions contains a pair of complementary literals P and $\neg P$.

Exercise 2.2: (4 P)

Prove Proposition 1.8 from the lecture.

Exercise 2.3: (2+1 P)

Let F and G be propositional formulas. Prove or refute:

- (1) If F , G and H are propositional formulas, $F \rightarrow G$ is valid, and $G \rightarrow H$ is satisfiable, then $F \rightarrow H$ is satisfiable.
- (2) If F is satisfiable and G is satisfiable, then $F \wedge G$ is satisfiable.

Exercise 2.4: (3 P)

Consider the following logical puzzle:

Four people (Prof. W., Mr. L., Mr. K., Mrs. D) are suspected to have committed a murder.

Is Prof. W innocent, then Mr. L is not suspected and the guilt from Mr. K would be undoubtful. Mr. K has an absolute secure alibi for the crime. Is Mr. L guilty, then is Prof. W also as well as Mr. K. Is Mr. K innocent, then is Mrs. D also innocent.

Who is the murderer?

Translate the sentences into propositional formulas, create the corresponding input file for the SAT Solver zChaff (<http://www.princeton.edu/~chaff/zchaff.html>) and try to answer the question.

Please send the zChaff input file electronically to your tutor.

Exercise 2.5: (2 P)

Use the transition system $\Rightarrow_{\text{DPLL}}$ for the DPLL procedure to check whether the following set of propositional clauses is satisfiable or not. For each step explain briefly which rule you use and why.

$$\begin{array}{rcll}
 A \vee B \vee C & & & (1) \\
 A \vee \neg C & & & (2) \\
 \neg A \vee D & & & (3) \\
 \neg A \vee E & & & (4) \\
 B \vee \neg D \vee \neg E & & & (5) \\
 \neg B & & & (6)
 \end{array}$$

Challenge Problem: (2 Bonus Points)

A Horn clause is a clause with at most one positive literal. Prove that $\Rightarrow_{\text{DPLL}}$ without the rule Decide is a decision procedure for Horn clause sets.

Submit your solution to Manuel Lamotte, building E14, office 115F until May 05, 8 am (office hours on workdays 7.30 - 15.00). Please write your name and the date of your tutorial group (Mon, Tue, Thu) on your solution.

Note: Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.