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Tutorials for "Automated Reasoning"
Exercise sheet 12

Exercise 12.1: (*4 P*)
Use superposition to show that the following set of (implicity universally quantified) clauses is unsatisfiable:

$$f(a,b) \approx a$$
$$f(a,x) \approx x$$
$$f(b,c) \not\approx c \vee a \not\approx b$$

Use the KBO with weight 1 for all function symbols and variables and the precedence $f > a > b > c$; compute only inferences that are required according to the ordering restrictions of the superposition calculus.

Exercise 12.2: (*3 P*)
Prove Lemma 4.57 from the lecture:

Let $N_0 \vdash N_1 \vdash N_2 \vdash \ldots$ be a run. Then $Red(N_i) \subseteq Red(N_\infty)$ and $Red(N_i) \subseteq Red(N_*)$ for every i.

Exercise 12.3: (*3+3 P*)
Consider the following clause set N:

$$(1) \qquad f(b,a) \approx a$$
$$(2) \qquad f(a,c) \not\approx b$$
$$(3) \quad f(x,y) \not\approx x \vee f(y,x) \approx x$$

1. Apply Superposition to saturate the clause set. Use KBO with weight 1 for all function symbols and variables and the precedence $f > c > a > b$. Hint: Delete tautologies and subsumed clauses during the saturation.

2. Construct the candidate interpretation R_∞ from all ground clauses generated by inserting the constants a, b, c for the variables.

Challenge Problem: (*2 Bonus Points*)
A Σ-interpretation $\mathcal{A}$ is called term-generated, if for every $b \in U_\mathcal{A}$ there is a ground term $t \in \mathrm{T}_\Sigma(\emptyset)$ such that $b = \mathcal{A}(\beta)(t)$. Prove: A set of (universally quantified) equational clauses has a model if and only if it has a term-generated model.

Submit your solution in lecture hall 002 during the lecture on July 10. Please write your name and the date of your tutorial group (Mon, Tue, Thu) on your solution.

Note: Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.