Universität des Saarlandes
FR Informatik

max planck institut informatik

Manuel Lamotte
Christoph Weidenbach

June 26, 2008

Tutorials for "Automated Reasoning"
Exercise sheet 11

Exercise 11.1: *(3 P)*
Apply the Knuth-Bendix completion procedure to the set of equations $\{\, f(f(x)) \approx x,\ f(a) \approx b \,\}$ using the Knuth-Bendix ordering with weight 1 for all function symbols and variables and the precedence $f > a > b$.

Exercise 11.2: *(3 P)*
Show that the inference rules *Pos. Superposition* and *Equality Factoring* of the superposition calculus are correct, i.e., show for each rule

$$\frac{C_n, \ldots, C_1}{C_0}$$

we have $\{C_1, \ldots, C_n\} \models C_0$.

Exercise 11.3: *(4 P)*
For each of the following term rewriting systems (1) to (8) give, if possible, an instance of a lexicographic path ordering and a Knuth-Bendix ordering such that the left-hand sides are larger than the right-hand sides when you compare the rules. If the left-hand side is not larger give a short argument why it is not.

$$
\begin{array}{rrcl}
(1) & f(f(x)) & \to & f(g(f(x))); \\
(2) & g(f(x)) & \to & f(h(g(x))); \\
(3) & g(f(x)) & \to & g(h(f(x))); \\
(4) & f(f(x)) & \to & g(x), \\
 & g(g(x)) & \to & g(f(x)); \\
(5) & h(g(x), g(x), y) & \to & h(x, y, y); \\
(6) & h(x, b, y) & \to & h(y, c, x); \\
(7) & h(x, f(x), y) & \to & h(g(x), x, g(x)); \\
(8) & h(x, y, y) & \to & h(b, y, f(y)).
\end{array}
$$

Challenge Problem: (*2 Bonus Points*)
Prove Lemma 4.39 from the lecture:

$$\text{If } E, R \vdash E', R', \text{ then } \approx_{E \cup R} = \approx_{E' \cup R'}.$$

Alternative Challenge Problem: (*2 Bonus Points*)
Show that there exists a reduction order $>$ that cannot be extended to a reduction order that is total on ground terms.

Submit your solution in lecture hall 002 during the lecture on July 03. Please write your name and the date of your tutorial group (Mon, Tue, Thu) on your solution.

Note: Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.