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June 19, 2008

Tutorials for "Automated Reasoning"
Exercise sheet 10

Exercise 10.1: (*1,5 + 1 + 1,5 P*)
Compute the set of critical pairs for each of the following systems:

$$R_1 = \{\, f(g(f(x))) \to x,\ f(g(x)) \to g(f(x)) \,\}$$
$$R_2 = \{\, f(x,x) \to a,\ f(x,g(x)) \to b \,\}$$
$$R_3 = \{\, f(g(x)) \to g(x),\ g(h(g(x), f(y))) \to f(x) \,\}$$

Exercise 10.2: (*2 P*)
Use the polynomial interpretation $\mathcal{A}$ with $U_{\mathcal{A}} = \mathbb{N} \setminus \{0,1,2\}$ and $P_f := X^2 + XY$ to show that the following term rewriting system terminates.

$$\begin{array}{rcl} f(f(x,y),z) & \to & f(x,f(y,z)) \\ f(y,f(x,z)) & \to & f(x,x) \end{array}$$

Exercise 10.3: (*2+1 P*)
Find a lexicographic path ordering that proves the termination of the first TRS and show that it is not possible to prove the termination of the the second rewrite system with a lexicographic path order.

(1)
$$\begin{array}{rcl} h(0,y) & \to & s(y) \\ h(s(x),0) & \to & h(x,s(0)) \\ h(s(x),s(y)) & \to & h(x,h(s(x),y)) \end{array}$$

(2)
$$\begin{array}{rcl} f(f(x)) & \to & g(x) \\ g(g(x)) & \to & f(x) \end{array}$$

Exercise 10.4: (*1+2 P*)
Find Knuth-Bendix orderings that prove the termination of the following term rewriting systems:

$$
\begin{array}{lrcl}
(1) & f(f(x)) & \to & g(x) \\
 & g(g(x)) & \to & f(x) \\
 & & & \\
(2) & s(x) + (y + z) & \to & x + (s(s(y)) + z) \\
 & s(x_1) + (x_2 + (x_3 + x_4)) & \to & x_1 + (x_3 + (x_2 + x_4))
\end{array}
$$

Challenge Problem: (*2 Bonus Points*)
Show that the Knuth-Bendix order is total on ground terms.

Submit your solution in lecture hall 002 during the lecture on June 26. Please write your name and the date of your tutorial group (Mon, Tue, Thu) on your solution.

Note: Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.