max planck institut
informatik

Universität
des
Saarlandes
FR Informatik

Uwe Waldmann
Christoph Weidenbach

June 14, 2006

**Tutorials for "Automated Reasoning"**
**Exercise sheet 8**

**Exercise 8.1:** (*2 P*)
Prop. 3.6 states that, if $\rightarrow$ is normalizing and confluent, then $b \leftrightarrow^* c \;\Leftrightarrow\; b{\downarrow} = c{\downarrow}$. Prove it by induction on the length of the derivation without using the Church-Rosser Theorem 3.3.

**Exercise 8.2:** (*2 P*)
Let $E = \{\, f(g(x)) \approx g(f(x)) \,\}$. Give a derivation for $E \vdash f(f(g(g(y)))) \approx g(g(f(f(y))))$.

**Exercise 8.3:** (*2 P*)
Consider the signature $\Sigma = (\{f, b, c\}, \emptyset)$ with $\mathrm{arity}(f) = 1$, $\mathrm{arity}(b) = 0$, $\mathrm{arity}(c) = 0$ and the set of (implicitly universally quantified) equations $E = \{\, f(f(x)) \approx x \,\}$. How many elements does the universe of $\mathrm{T}_\Sigma(\emptyset)/E$ have? How do they look like?

**Exercise 8.4:** (*2 P*)
Prove that $s \rightarrow_E t$ implies $E \vdash s \approx t$ by induction on the depth of the position where the equation is applied. (This is the first step of the proof of Lemma 3.11.)

**Exercise 8.5:** (*2 P*)
In the proof of Lemma 3.14, one has to show that $[r] = \mathcal{T}(\gamma)(r)$ for all terms $r \in \mathrm{T}_\Sigma(\{x_1, \dots, x_n\})$. Prove this statement.

Submit your solution in lecture hall 003 during the lecture on June 21. Please write your name and the date of your tutorial group (Mon, Thu, Fri) on your solution.

**Note:** Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.