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Tutorials for "Automated Reasoning"
Exercise sheet 5

Exercise 5.1: *(4 P)*
Prove the validity of the formula

$$(\forall x\, (p(x) \rightarrow p(f(x))) \wedge p(b)) \rightarrow \exists x\, p(f(f(x)))$$

via a ground resolution refutation. First, build the relevant CNF, second generate the necsssary ground clauses for the third, final ground resolution refutation.

Exercise 5.2: *(3 P)*
Let $\succ$ be the total ordering on the ground atoms $D \succ C \succ B \succ A$.

a) Determine the lexicographic ordering relationship between the three pairs $((A, A, B)$? $(A, B, A))$, $((C, A, B)$? $(B, D, D))$, $((B, A, A)$? $(A, B, B))$ generated by the lexicographic extension of $\succ$.

b) Consider the triples of a) as multisets and determine the relationship generated by the multiset extension of $\succ$.

Exercise 5.3: *(3 P)*
Let N be the clause set $\{A \vee B \vee C, \neg A \vee \neg B, \neg B \vee \neg A \vee C, \neg C \vee B\}$ and $\succ$ as in Exercise 5.2.

a) Determine I_N.

b) Which clause is false in I_N?

c) Show the resolution step yielding a smaller counterexample.

Challenge Problem: (*2 Bonus Points*)
Present an unsatisfiable propositional clause set where the shortest $\Rightarrow_{DPLL}$ refutation (considering the standard backjump clause over the decision literals) is longer (counting the number of fail and backjump steps) than the shortest refutation by resolution (counting the number of generated resolvents).

Submit your solution in lecture hall 003 during the lecture on May 29. Please write your name and the date of your tutorial group (Mon, Thu, Fri) on your solution.

Note: Joint solutions, prepared by up to three persons together, are allowed (but not encouraged). If you prepare your solution jointly, submit it only once and indicate all authors on the sheet.